Autonomous Quanta

On the Dynamism of Nature and the Nature of Dynamism

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This book is dedicated to the memory of
John Archibald Wheeler (1911-2008)
humane scientist of the highest order

No phenomenon is a physical phenomenon until it is an observed phenomenon.

J. A. Wheeler, 1979

The question, 'Can the quantum-mechanical description of physical reality be considered complete?,' has a positive answer. However, reality may be different for different observers.

A. Peres, 2004

Covering-law theorists tend to think that nature is well-regulated; in the extreme that there is a law to cover every case. I do not. I imagine that natural objects are much like people in societies. Their behaviour is constrained by some specific laws and by a handful of general principles, but it is not determined in detail, even statistically. What happens on most occasions is dictated by no law at all.

Nancy Cartwright, 1983

Preface

Physics predicts a dead bird thrown into the air follows a parabolic arc. A live bird thrown into the air goes where it will. And this but one of countless examples of the limits of physics to fully grasp nature. Why does physics rule so authoritatively in deep principles of symmetry and robust conservation laws yet describe so little of the richness of the natural world? The *Theory of Autonomous Quanta* is a response to this fundamental question.

Autonomous Quantum Theory brings to modern Quantum Theory the missing dynamism of *agency*, as understood in biology, economics, computation, and other sciences for which information and the action information triggers are fundamental.

Generalized position and momentum coordinates q, p originating in the canonical theory of Classical Mechanics are transfigured into density operators in Quantum Mechanics. Quantum operators, seated in and energized by elementary force fields, are revealed to be the most elementary agents of nature with the power to observe and move one another according to their observations. *Observations* in this generalized sense are force-field interactions that exchange information between agents.

Each of the canonical agents q, p holds the information of its own dynamic. A Hamiltonian superagent $\mathsf{H}(\mathsf{q}, \mathsf{p})$ holds the information of the state of the community of agents as a whole, encoded in their energy. Canonical agents observe the Hamiltonian information that directs the action to be executed by the time-evolution operator—producing the resulting motion of each agent. Quantum agency thus completes J. A. Wheeler's celebrated principle, "no phenomenon is a physical phenomenon until it is an observed phenomenon" with a co-principle, "Observations generate the motion of observers."

Autonomous quantum agents, as in evolution at higher levels of complexity, evolve from both definite and random data. Some random data according to the Heisenberg Uncertainty Principle is inevitably present in the observed Hamiltonian directing each agent's motion. This breaks unitary invariance. On the other hand, the Hamiltonian superagent evolves unitarily preserving the overall order of evolution while accommodating the unpredictable creativity of canonical agent motion.

The *theory of motion* began in earnest with Isaac Newton who first grasped the Law that governs the motion of the large-scale solar system. Pierre Louis Maupertuis, Joseph Louis Lagrange, William Rowan Hamilton, and others elaborated Newton's Laws into a refined and elegant *Theory of Classical Mechanics*. They showed that motion issued from a master principle in nature: the *principle of least action* for which a Hamiltonian function embodying the energy of the system played a central organizing role.

Werner Heisenberg advanced the Classical Theory of Motion initiated by Newton into the quantum realm of atoms, molecules, and other elementary particles. Heisenberg saw that generalized position and momentum coordinates q, p originating in the canonical theory of Classical Mechanics could be understood as operators—matrices rather than points—in Quantum Mechanics. The points of classical mechanics became points in the spectra of quantum operators. Quantum Theory became the more comprehensive theory of mechanics reducing to Newtonian theory in the limit in which *action* is large compared to the fundamental unit of action, the Planck quantum.

The quantum epitomized in Heisenberg's operator mechanics—and in its implementation in the Schrödinger equation—has been elaborated into a comprehensive Quantum Theory over the better part of the past century. Modern quantum theory is the success story that explains the Periodic Table of the Elements, reveals the atomic and molecular structure of matter, and underlies the electronics and computer chip revolution. However, something is missing.

John Archibald Wheeler puzzled over the following thought experiment:

> Paper in white the floor of the room, and rule it off in one-foot squares. Down on one's hands and knees, write in the first square a set of equations conceived as able to govern the physics of the universe. Think more overnight. Next day put a better set of equations into square two. Invite one's most respected colleagues to contribute to other squares. At the end of these labors, one has worked oneself out into the doorway. Stand up, look back on all those equations, some perhaps more hopeful than others, raise one's finger commandingly, and give the order "Fly!" Not one of those equations will put on wings, take off, or fly. Yet the universe "flies." Wheeler [17]

When I first encountered Wheeler's dictum that "no phenomenon is a physical phenomenon until it is an observed phenomenon" It dismayed me. It seemed to fall into that idealized tradition of quantum theory in which the human mind was directly involved in quantum processes. According to this line of thought, the collapse of the wave function in some quantum process being observed by a human being took place in the brain of the human being. The Wheeler dictum is difficult to abide if one regards observers as human beings in a macroscopic world. But If observers are nanophysical agents activated by the force field whose observations are more properly understood as signals exchanged with other nanophysical agents, a compelling picture emerges.

Wheeler's thought experiment focused on that part of nature represented in its computational or informational aspect: *governing equations*. Nature's hardware or energy presence in the force fields which animate those equations was not his focus. Both are present. The four well-established forces range from gravity to the strong nuclear force with a fifth possible to account for dark matter. Situated in between in strength is the electromagnetic force, the source of motion and structure of the abundant world of chemistry. Electromagnetism energizes electrons, protons, atoms, molecules, and ions, the force most intimately associated with life.

The Theory of Quantum Automata recognizes quantum observables as nanophysical agents of change and quantum information as the medium of observation and direction of their actions. Quantum agents are energized by elementary force fields and endogenously directed in time-evolution by their observed Hamiltonian. The Hamiltonian encodes the actions to be executed. Some random data according to the Heisenberg Principle is inevitably present in the observed Hamiltonian.

Autonomously evolved quantum agents, as in evolution at higher levels of complexity, evolve from both definite and partly random data. The quantum origin of randomness can be understood as the ultimate source of randomness in the universe [1]. Random quantum fluctuations then play out in macroscopic ways at higher emergent levels of nature beyond the microphysical. This does not mean the causes of macroscopic randomness from the random fluctuations of hyperion to the unpredictable stock market are immediately quantum. At higher levels of emergent complexity, macroscopic agents can be limited in their information acquisition by simple observation and memory incapacities rather than by Heisenberg uncertainty. They accordingly observe and act with some random data stemming from macroscopic limitations that are not directly or proximately quantum. But the indirect cause remains ultimately quantum.

Agent actions would be utterly deterministic save for the fact that the observation of the Hamiltonian code is tinged with randomness that foreshadows the way DNA code is tinged with randomness. The motion is therefore executed with some random elements. From a perspective of deterministic mechanical perfection, this is a mistake. Alan Turing perceptively declared that "making mistakes is a sign of intelligence." From an evolutionary perspective, echoing Turing, randomness is mechanical magic that opens the way to life and birds that fly where they will.

Introduction

The theory of Quantum Automata introduces agency into quantum physics and a fundamental new object bearing that agency: the *qubit automaton*. The qubit automaton is an *ur-element* of the dynamism of nature with a status comparable to that of the atom in the chemistry of matter. Each qubit automaton corresponds to a single degree of freedom, the elementary dimension of mechanics. Physical particles, such as atoms and molecules can be observed and acted upon by physicists in laboratories. Qubit automata observe and act on and among *themselves*.

The qubit automaton is built of familiar mathematical objects. It is composed of a pair of independent Hilbert-Schmidt operators in the operator space of the qubit—the most elementary quantum system possible. The qubit automaton is the quantum manifestation of the generalized position-momentum pair—canonical coordinates—of a classical mechanical degree of freedom. The way of understanding these operators as *quantum agents* which autonomously observe and act on their observations is fundamentally new.

An *agent*, in the present context, is a physical entity capable of sending and receiving information and acting on that information according to an internal code. Endowed by the force field with the ability to send and receive signals and act on them, qubit automata are the most elementary agents of nature. A quantum agent receives information about other agents according to its observations of a Hamiltonian superagent and acts on that information by executing an appropriate move directed by the Hamiltonian in the time-evolution operator. In this manner, the second critical ingredient of Hamiltonian mechanics makes it's appearance on the quantum level: the Hamiltonian superagent. Generalized position-momentum pairs constitute the individual agents of a quantum system. The collective population of all agents is embodied in the Hamiltonian superagent, a superposition of the energy contributions of the canonical coordinate agents.

Quantum agents are not omniscient: they do not have access to the global information state of all agents but only the information they obtain from their observations of the Hamiltonian superagent. Because of the Heisenberg Uncertainty Principle, some of their observational data will be random and their resulting motion will be a consequence of some random data.

The Theory of Quantum Automata is founded on three postulates: *existence, observation*, and *evolution.*

Existence

The existence postulate proposes that elementary force fields sustained by physical charge manifest themselves as *quantum agents.* Quantum agents are nanophysical machines with both a hardware and software presence. Force fields are agent hardware. Hilbert-Schmidt operators (also known as 'quantum observables') are their software.

Quantum agents are canonical: they recapitulate the structural organization of Hamiltonian mechanics. Each degree of freedom is described by a pair of agents as canonical coordinates, a generalized 'position' and 'momentum'. The state of a system of N freedoms is thereby fixed by $2N$ agent-state coordinates. In the classical case, these are functions. In the quantum case, they are operators capable of both holding state and of inducing motion in other agents as well as themselves.

Each agent-state is mathematically a density operator and is postulated to hold the information of its own state of being. Each agent-state possesses a set of registers composed of the agent's eigenvectors which project to singleton states. An agent's state of being is stored in its registers as the probability the state represented by the register is occupied.

Observation

A key principle of Automata Theory is that agents are not omniscient. Each canonical agent holds its own state of being *but it does not hold the states of other agents* contrary to the notion of the 'state function' in conventional quantum mechanics. Information is exchanged between agents as signals transmitted and received via the elementary force field. An agent in Quantum Automata Theory only gains information about other agents dynamically in the course of interaction with them.

Individual state information is stored in the registers of each agent-state. This state, stored in its registers, is distinctly that of the agent—its *Hamiltonian-generated state* at each instant of time. The global state of the community of agents as whole is stored in the registers of a Hamiltonian agent which is a superposition of the energy contribution of each canonical agent to the whole.* In recognition of its pre-eminent global role, the Hamiltonian is regarded as a *superagent.*

*This global function originates in the principle of least action from which the motion of the system flows from a single master principle: that action be an extremum.

The Hamiltonian superagent is a sum of individual agent energy contributions to the total energy and thereby a superposition of agent states without distinguishing individuals. On the other hand, the canonical coordinate agents hold individual states of being but they lack information about the state of the system as a whole. That information is held by the Hamiltonian. Interaction brings agents and superagent together, dynamically evolving the system. The interaction of a canonical agent with the Hamiltonian superagent is simultaneously an observation of the Hamiltonian by that agent and the dynamic event that advances the agent's motion by the time-evolution operator to its "new state in time."

I incorporate the Born rule as the principle that describes agent observations (though more refined rules of observation may be accommodated within the automata framework). Agents gain information about other agents in observation of them in their collective presence in the Hamiltonian. As no division is made between "system and environment," the Hamiltonian is a time-invariant of the whole.The agent-observed Hamiltonian is not in general the same as the time-invariant Hamiltonian. Random bits enter the observation that distinguish it from the Hamiltonian agent's time-invariant state.

The observation postulate codifies Asher Peres's proposal [11] that different observers do not necessarily observe the same state of a given observable. This postulate endows agency theory with *quantum contextuality* [10, 19, 20]. Only if the agent and the Hamiltonian superagent share the same context will the agent observation produce the Hamiltonian state. Agents, sharing the same context, commute.

Evolution

Quantum evolution incorporates *observation* and *motion* in a single dynamic event. Individual agents observe the Hamiltonian and move according to directives computed from those observations as input to the time-evolution operator. Observation in automata theory is intrinsic to the dynamic of motion. Observation and motion constitute a unified dynamic process.

Quantum evolution acts to advance the time-evolution of agents based on its received encoding of the individual agent states in the Hamiltonian. That encoded Hamiltonian directs the time-evolution operator (in concert with the initial data of the present states) in new, time-advanced agent-states.

I accept the general time-evolution operator of quantum theory for which the Hamiltonian is the generator of the motion of each agent. In conventional quantum theory the Hamiltonian is assumed omnisciently present to all observables of the theory. This is not so for autonomous quanta in the present theory for which the Hamiltonian directing the evolution of each agent in the time-evolution operator is the Hamiltonian observed by that agent.

A major implication of the observation and evolution postulates is that information is not universally shared among the nanophysical actors in a quantum system. This leads to a certain independence among the agents in their dynamic evolution, an opening for the emergence of random information—and the novelty that flows from it in the subsequent evolution of the system.

In addition to the postulates, several ancillary principles emerge that support or are implied by the postulates and the experimental history of quantum mechanics.

Chapter 1

Dynamism

Elementary forces of nature charge the universe with *dynamism*. * Driven by its internal force fields, matter is in motion, from the impulse of shock waves of colliding galaxies to the microscopic quantum jiggling of carbon and nitrogen atoms in DNA and the proteins of life.

Some states of matter may be strongly stable—seemingly motionless—over long periods of time, such as the proton. Other states have vanishingly small lifetimes before being swept away into to new ones such as the chaotically thrashing position and momentum states of atoms in the hot, dense gases of stars. Between these extremes are vast ranges motion. Included among them are planetary scale objects typical of the earth, living creatures, and humanity.

> The dynamism of nature springs not from some external source or agency, nor from gods or demiurges standing outside the natural world, but from natural forces within. These are the four fundamental force fields of gravity, the weak-nuclear force, the electromagnetic force, and the strong-nuclear force.† The fields are sustained by *charges* indigenous to matter that are seats of force. Mass is the charge sustaining the force of gravity. Electric charge is seat of force in protons and electrons or binding them into electrically neutral atoms. The proton itself is a bound particle comprised of eight color-charged quarks sustaining the strong nuclear force. Neutrinos, companion particles to the electron, are endowed with hypercharge sustaining the electro-weak force. These forces push and pull particles into chemically bound configurations, or if energy levels are greater than bonding levels, break bonds and scatter particles.

*It isn't possible to give a fully satisfactory definition of a fundamental process like *dynamism* in terms of other concepts. It's fundamental! Some insight can be gained from the definition: dynamism is the process whereby elementary particles observe one another and act of those observations according to a rule imposed by the law of motion.

†A fifth force is possible, principally to account for dark matter.

The mass-energy of force fields makes the universe be and move on scales from microscopic to cosmic. Mass-energy provides the raw material for being and motion. Information shapes that raw material by specifying the state which matter adopts at any time and the state to which it will move. Energy and information are twin principles at the foundation of matter and motion

Dynamism includes the classical motion we observe at the human scale of baseballs and orbiting planets of the solar system. But below the surface of the regularity of large-scale motion is the spontaneity of the quantum which continuously streams up from its microscopic depths in the atomic and molecular chaos of gases to buzzing Brownian agitation in liquids, to rare random bond flips in well-ordered macromolecules.

Life is the outstanding example of dynamism, but not all dynamism is life. Beyond life, the geological structures of the planet send and receive seismic signals which in turn induce tectonic motion. Before life, the physics and chemistry of organic and amino acids and amphiphilic compounds were sufficiently evolved to self-assemble bilayers separating one set of chemicals from another—an important pre-curser to the cell. Inorganic nature is rich in self-organization in which simple atoms and molecules interact through their force fields to create complex abiotic chemical cycles and structures: dynamism outside of life, much of which is a prelude to life.

To understand the dynamism of nature, abandon images of quantum particles as infinitesimal billiard balls or even as minuscule clouds of electrons polarized around protons. One must instead appreciate elementary particles as submicroscopic machines with internal parts that move and act and whose behavior is regulated by natural computer code embedded in their quantum machinery we know as laws of motion.

The Spontaneous Quantum

Endless forms most beautiful foliate the universe from the hydrogen and helium clouds of the primordial big bang to the billions of complex protein molecules that sustain the tissues and organs of life. A 'state' of matter is one alternative—*a way to be* of the many ways for matter to be. Water can be ice, liquid, or steam. Large scale ways to be of trillions of atoms and molecules—such as water as steam rather than as ice or liquid—are composed of the ways to be of more elementary components and their ways to be.

Steam is composed of water molecules with sufficient energy to collide without clinging together and condensing as liquid or ice: a gas. A water molecule in turn is composed of Oxygen and Hydrogen atoms, each with ways to rotate and vibrate within the molecular bonds that hold them together. Atoms in turn are clouds of electrons bonded to nuclei each with yet more elementary ways to be. The atom's electrons have a multitude of alternative orbits of ascending energy levels bounded by the lowest possible state, the ground state. At the finest level, an individual electron has two intrinsic ways to spin—clockwise or counterclockwise: two spin states. There is no finer way to be than in one or the other of two states. Two ways to be: the limit.

The complex world of all the ways to be are composed from an elementary way to be: a pair of alternatives, two states: the *bit.* At bottom, the universe is one bit. At top, the accessible universe is estimated to contain 10^{120} bits in all its matter and energy with a corresponding exponential explosion of possible states.

Motion is change. The primal unit of change in motion is also the bit: a move from one states to another The light switch on the wall has *two* states: "off" or "on." Try to reduce the light switch to *one* state: it is always off or always on—no longer a switch, no longer capable of change, no longer capable of motion. Half a bit is no bit.

The bit is exclusive. A switch is either on or off, not both at once. The bit is binary, holding any representation of two alternatives: On-off, up-down, right-left, yes-no, true-false. . . . The bit: the manifestation of elementary logic in nature.

You are watching an ordinary light switch. The switch is off. Suddenly it flips and the lights come on. You stare in amazement. Who did that? And as you continue to stare, the switch without warning flips again and the lights just as mysteriously go off. What is going on? A loose wire? A mouse in the electrical box? You continue to watch and again, another flip, but this time more quickly than the first. A long time passes and you think it might stay on, but no, it flips off again. Sometimes there are long runs of off or on, at other times several on- or off- off bursts in a row. The pattern is not predictable. It has random elements. This light switch is not a bit. It is a quantum bit or *qubit*: a quantum light switch.

The bit is predictable. The qubit is not. The bit is exclusively on or off. The qubit can be both at once—a suspended state of animation known as the celebrated quantum superposition. It remains in superposition as long as it is not disturbed. When it interacts with another qubit, possibly also in superposition, the interaction can stimulate the superposition to transition from qubit to bit. In that transition, nature does not ordain whether the qubit emerges as an 'off bit' or an 'on bit.' Nor does nature reveal beforehand the precise moment the transition will occur. The final state of a spontaneous transition from superposition to bit and the moment it might occur are both not completely predictable.

Imagine now the quantum light switch reverts to ordinary, flipping on and off in a regular pattern, like a blinking stop light. The qubit's life-like quality vanishes. A blinking light switch, or even more complex but regular on-off patterns (such as a Broadway theater or sports arena jumbo-tron billboard) reveals itself as humanly designed, humanly caused. It is not by accident that the quantum light switch, like life, fascinates us with its unpredictable, manifold possibilities, but the bit provokes no sense of awe, no suggestion that it is alive. Stripped of its quantum nature, the bit exhibits the inevitable—often boring—motion we call 'classical.' The quantum light switch intimates life. A blinking stop-light does not.

To directly experience quantum spontaneity, one must descend to a scale more than a billion times smaller than the switch on the wall and the finger that flips it, the world of the nanoscale. There, one encounters quantum objects—atoms, molecules, electrons, photons—spontaneously jiggling and flying about whose raucous motion is a blur at the large scale. There. an atom separated between two valleys by a mountain peak of energy can be trapped in one or the other of the valleys. If the atom were a massive body, like a marble rolling around at the bottom of one of the valleys, it could never explore the other valley. It doesn't have enough energy to roll up and over the peak. But a quantum particle can be in a superposition of both valleys at once. It spontaneously tunnels back and forth through the energy mountain from one valley to the other—and the moment it will do so is not precisely predictable.

Experience quantum randomness in your own nervous system. Draw an arrow into a bow and pull back. A small but persistent unsteadiness invades your hands and arms as you draw the string taut to a precise aim. Archers strive to overcome this trembling, but it cannot be completely eliminated because our nervous systems are not completely ordered in regular motion. The proteins and energy transporting molecules that sustain nerve and muscle are nanoscale quantum machines beset with random fluctuations ultimately originating in quantum uncertainty. The fluctuations are small compared to the directed motion the brain commands, but they are not absent. They make themselves known in the quivering that makes it impossible to precisely hold the arrow on the line of sight to the target.

Carnival ride bumper cars are massive objects weighing hundreds of pounds. The trillions and trillions of bits in atoms and molecules that compose them jiggle spontaneously at the quantum level. Bumper cars, like billiard balls, bounce off one another unstably. Atomic and molecular collisions magnify slight differences in trajectories and multiple collisions magnify these differences exponentially. Small quantum fluctuations too are subject to the exponential magnification of unstable collisions. Half a dozen collisions or more are sufficient to boost quantum fluctuations jiggling in the atoms and molecules of a bumper car to the human scale producing a slightly different trajectory than would have occurred without the quantum effects. This difference is measurable in millimeters, not nanometers thanks to the magnifying power of less than a dozen collisions [1]. Atoms and molecules too collide unstably in a gas and in a more complex manner in liquids with similar exponential magnification of quantum random outcomes of the encounter.

The Regular World: Law of Motion

The quantum spontaneity of the universe is expressed in a law of nature, the Heisenberg Uncertainty Principle. This outstanding law declares 'there is no law.' This is not a paradox. We can more precisely state 'there is no law that completely fixes the outcomes of every physical interaction, every dynamic event.' It's the law. But much of the outcome is lawfully ordered and predictable. Nature is neither inevitably random nor completely lawful and predictable. Quantum spontaneity is only one-half the story.

The other half is the regularity and predictability of the universe from the steady orbiting of planets (we use the rotation of the earth to mark our basic unit of time: 1 day) to the steady beating of hearts. While the quantum is about unpredictable spontaneity, the Law of Motion is all about predictable regularity. The Law of Motion captures the continuity and stability of motion while the quantum nips at it heels injecting* wisps of novelty and new possibilities into every change of state.

Think of the universe as fabric woven on a loom. The warp threads are the laws of motion—rigid, and invariant, the weft, the emergent random strands that weave within the ordered warp. Together order and randomness form a creative whole.

Dynamic Principles: Energy and Information

Who or what flipped the qubit switch? Answer: that fundamental source of action that pervades all of nature: energy and information. Energy makes the universe be and go. Information tells it what to be and where to go. Energy provides the raw potential for being and motion. Information is the data that shapes that raw potential by specifying the state in which matter will be and the state to which it will move.

The combination of energy and *information* are hallmarks of the quantum nature of matter. The uniquely quantum nature of the dynamic is that quantum dynamics produce both definite new states highly correlated with existing states *and* spontaneously random new states. The bit of an ordinary light switch is activated by energy, but the activation is not spontaneous. It required your energized finger to flip the switch. The qubit of a quantum light switch is also activated by energy. But the activation can be spontaneous: the qubit of a quantum particle, energized by its underlying force field can, within the flow of its deterministic motion, spontaneously flip itself.

Chapter 2

Duality of Matter and Information

Aristotle had a sense of nature that prefigured quantum discoveries. He proposed that all the objects in nature were composed of two ingredients—prime matter and substantial form. Prime matter was material stuff, but it was infinitely fungible. A favorite metaphor of Greek cosmology was wax which could be molded and remolded into arbitrary shapes. It wasn't any particular thing but had the potential to be any material thing whatsoever: some-thing. Substantial form endowed prime matter with that some-thingness. Aristotle emphasized that the pair were inseparable. One did not find pure matter or pure form cleaved from one another. Form was embodied in matter. Matter was embodied form.

Aristotle's prime matter lines up with our modern notion of mass-energy—recall that since Einstein, mass and energy are different forms of the same thing *mass-energy*. Aristotle's form lines up with the modern notion of information. Information is the data contained in strings of bits. The bits aren't the matter. The bits are matter's information content. The information conveyed in this sentence is encoded in the bits of computer chip transistors as I write. Those material bits are not the information contained in the sentence. They are hardware switches. But where then, you ask, is the information? It is encoded in the switches (See Chapter

Bits don't reside in airy nothingness. Bits are encoded in matter from the toggle of a light switch to the positions of nucleotides in a DNA molecules. it isn't until bits are given values that an information string becomes *some-thing*. And when the bits are changed, that something becomes some-thing else. In the modern idiom of computer science, Aristotelian prime matter is *hardware*, substantial form, *software*. Hardware can't compute without software. Software has no existence outside hardware. Even non-electronic software such as code printed or hand-written on a page is bound to the hardware of paper and ink.

Bits reside in matter—in the spin states of an electron, the polarization states of a photon, the position states of an atom oscillating between two potential wells, or the atom-atom bonds of a molecule. The information change of the bits is reflected in a change of physical structure—in a change in the matter in which the information is embodied whether spin state, a new position in a potential, or a different shape of a molecule. At the quantum level, the information bits are qubits—until interactions transition them to bits. The nitrogen atom hovering above the plane formed by three hydrogen atoms of an ammonia molecule represents the up-state of one of the qubits describing the Nitrogen's position. The Nitrogen spontaneously tunnels across the plane of the hydrogens as it interacts with them and now hovers below in the down state. A bit has flipped—a quantum switch in the form of an ammonia molecule.

Teleportation dramatically illustrates Aristotelian matter and form. A physical particle at one location (such as an atom in a certain energy state) remains at that location while its physical state—its quantum state existent as quantum information—is entangled with a pair of qubits and sent as pure information via these information bearing particles, typically photons, to a distant location where an atom at that location is activated with the transmitted information to reproduce the original energy state. In the process, *the state of the original atom is destroyed once the information resides in its new location.*

Do observe that while the state is being teleported, say via photons, the information resides in the material medium of the photons: in charge-conductions sensors, in conduction electrons in wires and other electronic components, then back to photons from electrons for the long trip in an optical cable, before it's final emergence in an inverse set of operations as the teleported state of the distant atom to reside as the state of atom at the new location. Information is inevitably coded in matter at each stage along the route.

The force fields—energy fields—of nature coupled with their quantum information fields are natural creative systems. The atom is a paradigm in which an astonishingly simple inverse-square Coulomb force marshaled by quantum information encoded in a Hamiltonian and executed by the Schrödinger equation spawns the richness of the periodic table of the elements.

> The theory of autonomous quanta brings together the quantum principle of superposition as an organizing principle of multiple contributors—qubit automata—according to the information they share. Superposition exists because quanta transmit their states of being into a common space-time according to a rule: the information that is their state of being is coded in energy for consumption by the community as a whole. Each qubit automaton is an element of the whole. The Hamiltonian holds both the energy and the information state of the community of qubits as a whole.

Chapter 3

The Computational Stance

Quantum particles interact, scattering in collisions or associating and dissociating in chemical bonds as complex particles.* The particle is sustained by the energy of its atomic and sub-atomic force fields and controlled by natural computer code embedded in the force field—its quantum information.

I say 'natural' because all nature, not just human beings, computes [Lloyd, 2006]. The elements of computation are states of matter. Every instant of motion of the atoms, molecules, and other particles of the universe is a computation in which the present state of matter is input and the next state in time is output. The elementary hardware of nature consists of atoms, electrons, photons, molecules and other particles whose interactions through their force fields sustain this natural computation. The software 'running' the natural microprocessors is the law of motion according to the operator algebra (Chapter 3) with which nature has endowed the force fields that impel the particles.

To understand how nature computes, appreciate elementary particles as nanophysical machines with internal parts that move and act and whose behavior is regulated by natural computer code embedded in their machinery. Humanly designed computation harnesses the native computation of atoms, electrons, molecules, and other elementary particles to the hardware of computer chip transistors which are physically embodied bits. This humanly designed microprocessor runs humanly designed software stored in its registers. This software is a higher level program running lower level natural software intrinsic to atoms and electrons composing electronic transistor gates to close and open resulting in computational moves.

Operator algebra in the quantum context can be regarded as reverse-engineering

*These range from gases to large-scale, tightly bonded condensed-matter assemblies and phases. Atoms emerge from nuclei and electrons, molecules from atoms, polymers, minerals, and other supermolecules from molecules, cells from supermolecules, and onward to all the complex matter of the inorganic and organic world.

of the natural computer language of quantum software. As a testament to the computational universality of nature and the power of operator algebra, very different hardware arrangements (atom in a double well, atom in a cavity, spinning electrons, atoms forming or breaking molecular bonds, etc.) all run the *same* software: quantum operator algebra described in Chapter 3.

Because of its generality, I shall cast my narrative in the *computational stance*, a universal computational representation for economy and clarity. Any quantum system can be described (simulated) to arbitrary precision by a quantum computational network in which qubits are the fundamental elements [Deutsch, 1981]. Quantum networks implemented in qubits may not physically be the particular quantum system they simulate, but computational universality is a sufficient foundation to support the general theory I shall put forth.

I adopt in my illustrations countable, non-relativistic systems of quanta that will, for the most part, be finite. These include typical collections of atoms, molecules, and other elementary particles where processes of quantum superposition, uncertainty, entanglement, decoherence, and randomness, salient to the dynamic, are pointedly in play.

Chapter 4

Quantum Automata

Qubits require physicists in laboratories to manipulate them. Qubit Automata manipulate themselves and one another.

...according to quantum theory, we are all made of algebra-stuff: the elements of local reality are faithfully described not by real variables or stochastic real variables but by the elements of a certain algebra that can be represented by Hermitian matrices. —D. Deutsch [3]

Every moment of being is a moment of becoming. Isaac Newton made this clear with his discovery that a particle's state of motion is specified not only by its position. The particle's *rate of change* of position—its momentum coordinate—is also necessary to fix the particle's current state. Each degree of freedom of a mechanical system supports a generalized position-momentum pair of coordinate. Position is the coordinate of being—"the being of being." Momentum is the coordinate of becoming—"the becoming of being."*

Nature presents us with both a quantum and classical presentation of position and momentum. Classically, these are a pair of *coordinate functions* in phase space. Quantum-mechanically, these are a pair of *operators* in Hilbert-Schmidt space. The link between the two is *observation.* Classical values are observed values. Observers can be physicists in laboratories conducting experiments; but observation, in the sense of Automata Theory, goes well beyond human observers. I shall argue that observations, such as those by humans (or other observation-capable organisms), is an instance of the process of *interaction* in which information is exchanged between agents. I shall refer to this generalized representation, both classical and quantum, as *canonical.*

*Thus did Newton gracefully dissolve the Zeno paradoxes of antiquity in which the motion of Achilles and the tortoise he could never catch were described with only their position coordinates.

Quantum states are represented by vectors. Corresponding to the space of vectors is a higher-dimensional space of operators, Hilbert-Schmidt space. Operators are represented by matrices. A d-dimensional vector space is a subspace of its larger d^2-dimensional Hilbert-Schmidt space. Operators, like vectors, possess a state of being in their matrix entries, but operators surpass vectors in also being capable of action. Operators act on vectors and other operators and change them. I hold that the proper space of both nature's states *and* action is Hilbert-Schmidt space. Operators provide quantum automata with their *automaticity*.

The action principle evolves particles from one state to the next under the direction of its total energy function, the *Hamiltonian*. The Hamiltonian is typically a sum of the kinetic and potential energies of the particle, these energies are functions of the positions and momenta describing the state. The dynamic of matter was thereby fully deterministic. The transition of a system of particles from one state to the next was determined by the initial state.

The classical law of motion describes a universe in which information fixing states, namely, the position and momentum data of the particles, is freely communicated among them and fully available to the Hamiltonian (or to an omniscient being who holds such data) that moves particles accordingly.

Operator Algebra for the Qubit $d = 2$

A single degree of freedom of a mechanical system has a rich structure in its quantum information as a *qubit*. Multiple degrees of freedom correspond to a system of multiple qubits.

The Hilbert-Schmidt space of a qubit is spanned by four operators. One of these is the identity and is invariant. Only two of the other three are independent as I shall shortly describe leaving two independent operators. The independent operators of the qubit form the generalized position-momentum pair of a single quantum degree of freedom.

The set of Hilbert-Schmidt operators for the qubit forms the most elementary nanophysical community of a quantum system—and indeed, of all nature. I shall refer to the set as the *qubit automaton* composed of three physical actors represented in software by the operators $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3$ and a logical operator $\boldsymbol{\sigma}_4 = \mathsf{I}$, the identity, as its internal parts. The identity does not execute motion but is necessary to the dynamics of actors as that actor which validates the identity of each agent.

Quantum operators are endowed with physicality by the condition that they be normed (they manifest the physical quantity they represent somewhere in the state space) and that they be Hermitian ($\boldsymbol{\sigma}^\dagger = \boldsymbol{\sigma}$): eigenvalues representing the observable or measurable values of these physical quantities are real numbers.

These conditions endow the operators with an algebra whose product rule is:

$$\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 = i\boldsymbol{\sigma}_3 \qquad \boldsymbol{\sigma}_2\boldsymbol{\sigma}_3 = i\boldsymbol{\sigma}_1 \qquad \boldsymbol{\sigma}_3\boldsymbol{\sigma}_1 = i\boldsymbol{\sigma}_2. \tag{4.1}$$

The matrix representations of these operators are 2×2 matrices. The first three operators are physical operators with vanishing trace:

$$\mathsf{Tr}\,(\boldsymbol{\sigma}_\mu) = 0, \quad \mu = 1, 2, 3.$$

The fourth is a logical operator, the identity with trace 2. (The unitary algebra for multiple qubits may be found in Appendix 1.)

Now a remarkable fact: the qubit is dynamically fixed in the unitary algebra by a pair of independent automata (rather than three) with the third dependent upon the independent pair as their product in Eqs. (4.1). The third machine is a composition of the two independent machines and fixed by them. (One can visualize the machines as moving parts connected with mathematical links so that when any two are put into definite states, they lock the third into a definite state as well.) In this role, the independent machines are coordinates fixing the state of the qubit. The three automata coordinates for a particular qubit are $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_3$.

One can arbitrarily label one of the independent operators as the 'position' coordinate which by convention can be taken as $\boldsymbol{\sigma}_3$ and its mate $\boldsymbol{\sigma}_1$ as the 'momentum' coordinate. The third coordinate $\boldsymbol{\sigma}_2$ is dependent upon them according to the unitary algebra as

$$\boldsymbol{\sigma}_2 = i\boldsymbol{\sigma}_1\boldsymbol{\sigma}_3.$$

Agency: Agent-Actors and Agent-States

The qubit's operator algebra recapitulates the canonical structure of Lagrangian and Hamiltonian mechanics with qubit automata as canonical coordinates. To show this, I shall re-label $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_3$ as position and momentum operators Q, P,

$$\mathsf{Q} \equiv \boldsymbol{\sigma}_3, \qquad \mathsf{P} \equiv \boldsymbol{\sigma}_1$$

The third—and now dependent—automaton as the product of the canonical pair $\mathsf{S} = i\mathsf{PQ}$ I shall call the "action" as it mimics the acton $\mathsf{S} = \int \mathsf{P} d\mathsf{Q} \sim \mathsf{PQ}$ for free classical motion.[1] The complete set of products of the algebra that cyclically reproduce one another is

$$\mathsf{Q} = i\mathsf{SP}, \qquad \mathsf{P} = i\mathsf{QS}, \qquad \mathsf{S} = i\mathsf{PQ}. \tag{4.2}$$

The algebra closes with the identity:

$$\mathsf{Q}^2 = \mathsf{I}, \qquad \mathsf{P}^2 = \mathsf{I}, \qquad \mathsf{S}^2 = \mathsf{I}.$$

The operators $\mathsf{Q}, \mathsf{P}, \mathsf{S}$ are agent-actors: they execute actions transforming state-vectors and other operators but do not hold state—they are traceless operators (not unity-trace density matrices) obeying the commutation relations

$$[\mathsf{Q}, \mathsf{P}] = 2i\mathsf{S}, \quad [\mathsf{P}, \mathsf{S}] = 2i\mathsf{Q}, \quad [\mathsf{S}, \mathsf{Q}] = 2i\mathsf{P}.$$

Agent-States

Corresponding to agent-actors $\mathsf{Q}, \mathsf{P}, \mathsf{S}$ are agent-states $\mathsf{q}, \mathsf{p}, \mathsf{s}$—unit-trace density operators that hold the state-of-being of the agent. Agent-states are rank-1 density matrices distinguishing pure states (they have purity $\mathsf{Tr}\left(\mathsf{q}^2\right) = \mathsf{Tr}\left(\mathsf{q}\right) = 1$ and similarly for the other two agents).†

$$\mathsf{q} = \frac{1}{2}\left(\mathsf{I} + \mathsf{Q}\right), \qquad \mathsf{p} = \frac{1}{2}\left(\mathsf{I} + \mathsf{P}\right), \quad \mathsf{s} = \frac{1}{2}\left(\mathsf{I} + \mathsf{S}\right). \tag{4.3}$$
$$\mathsf{Tr}\left(\mathsf{q}\right) = 1, \qquad \mathsf{Tr}\left(\mathsf{p}\right) = 1, \qquad \mathsf{Tr}\left(\mathsf{s}\right) = 1.$$
$$\mathsf{q}^2 = \mathsf{q}, \qquad \mathsf{p}^2 = \mathsf{p}, \qquad \mathsf{s}^2 = \mathsf{s}.$$

The unitary algebra of agent-states is readily derived from their actor presentation as

$$\mathsf{q} = i[\mathsf{p}, \mathsf{s}] + \frac{1}{2}\mathsf{I}, \qquad \mathsf{s} = i[\mathsf{q}, \mathsf{p}] + \frac{1}{2}\mathsf{I}. \qquad \mathsf{p} = i[\mathsf{s}, \mathsf{q}] + \frac{1}{2}\mathsf{I}. \tag{4.4}$$

Agent-state products (4.4) do not mimic agent-actor products in the unitary algebra but rather as the commutator product and identity.‡

Agent-actors have a corresponding inverse relationship to the state presentation.§ The map from the state presentation to the actor presentation dual to (4.3) is

$$\mathsf{Q} = 2\mathsf{q} - \mathsf{I}, \quad \mathsf{P} = 2\mathsf{p} - \mathsf{I}, \quad \mathsf{S} = 2\mathsf{s} - \mathsf{I}.$$

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†The relationship of agent-states to agent-actors (4.3) has an arbitrary global phase. Agent-states could have been represented as $\mathsf{q} = \frac{1}{2}(\mathsf{I} + e^{i\phi}\mathsf{Q})$ with $\phi = \pi$ constituting a 'flip' and similarly for the other agents. However, once a a global phase is fixed, the system will evolve in time—agent -states and agent-actors q, Q and the others traverse values consistently related at any time to those of earlier or later times.

‡Some additional useful relationships between states and actors:

$$\mathsf{Q} = 2i[\mathsf{p}, \mathsf{s}], \qquad \mathsf{P} = 2i[\mathsf{s}, \mathsf{q}], \qquad \mathsf{S} = 2i[\mathsf{q}, \mathsf{p}],$$
$$[\mathsf{P}, \mathsf{s}] = i\mathsf{Q}, \qquad [\mathsf{P}, \mathsf{q}] = i\mathsf{S}$$

§The dual maps from actor presentation as $d \times d$ operators $\mathsf{Q}, \mathsf{P}, \mathsf{S}$ acting on d-dimensional state-vectors to state presentation as $d \times d$ density matrices $\mathsf{q}, \mathsf{p}, \mathsf{s}$ in a d^2-dimensional space are similar to Choi-Jamiolkowski maps.

One can regard agent-states and agent-actors as different organs of the same agent distantly analogous to biological neural systems holding a state of action in the motor cortex on the one hand and motor proteins executing muscle action of the same organism on the other.

A state-vector has no capacity for action: it is a helpless object. Outside agents (notably demonstrated by physicists in laboratories) supply the missing capacity by their arrangement of experimental configurations and their execution of experimental protocols that manipulate state-vectors.

Quantum operations performed by agents from the environment external to a system are an essential part of conventional open systems quantum theory. Quantum Automata Theory accommodates such external agents, but they are not essential as no division fundamentally exists between systems and their environment.

Primacy of the Position-Momentum Pair

The position-momentum pair of agent-states for each degree of freedom

$$\mathsf{q}, \mathsf{p}$$

is a foundational property of mechanics flowing from the principle of least action (See Chapter 8). These are "generalized coordinates" not restricted to translations and may range over other degrees of freedom such as rotations and spin.

One of the relationships found throughout both classical and quantum mechanics is the link of position and momentum through the Fourier transform. Take any non-degenerate state in the Hilbert space of agent-states (unit-trace density matrices), not necessarily a Pauli state. Designate it the "position" state q. Take its Fourier transform by the unitary transform F. The result is its canonical mate p, the momentum. Position is likewise the Fourier transform of the momentum. They are inverse to one another:

$$\mathsf{p} = \mathsf{F}^{-1}\mathsf{q}\mathsf{F}, \qquad \mathsf{q} = \mathsf{F}\mathsf{p}\mathsf{F}^{-1}.$$

Moreover, the dependent coordinate s, as already indicated (See Eq. (4.4)), is fixed by this position-momentum pair. So the fundamental coordinates of mechanics exist for any typical state in the space of states and the Fourier transform. The position-momentum pair so produced fulfill all the relationships (4.3) - (4.4)

Superposition Principle

Wholistic Nature of Quantum Dynamics

Quantum Motion is communal. Quantum dynamics describes the time-evolution of a superposition of states rather than the time-evolution of an individual state. Superposition is a linear operation in which states are added together with coefficients that describe their correlations.

Principle (Superposition). *Let* $\{|\mathsf{a}, j\rangle, j = 1, 2, \ldots d\}$ *be the set of eigenvectors which compose an agent-state* $|\mathsf{a}\rangle$ *with superposition coefficients* $\sqrt{\mathfrak{p}_{\mathsf{a},j}}$*. I anticipate that these coefficients are "pre-probabiliities," square-roots of probabilities,*

$$|\mathsf{a}\rangle = \sum_j \sqrt{\mathfrak{p}_{\mathsf{a}j}} |\mathsf{a}, j\rangle.$$

Note that pre-probabilities $\sqrt{\mathfrak{p}_j}$ *are complex amplitudes: their square magnitudes (Hermitian norm) are real number probabilities*

$$\mathfrak{p}_j = (\sqrt{\mathfrak{p}_{\mathsf{a}j}})^* \sqrt{\mathfrak{p}_{\mathsf{a}j}}.$$

Individual states emerge from the superposition in acts of observation intrinsic to the dynamic of motion (Observation as Motion, Chapter). The superposition transitions to a probability distribution under the action of observation. The observation is registered in the next state to which the agent is dynamically directed by the observed Hamiltonian.

As was pointed out earlier, The density matrix of this superposition of pure states is the self-outer-product of its state-vector

$$\mathsf{a} \equiv |\mathsf{a}\rangle\langle\mathsf{a}| = \big(\sum_j \sqrt{\mathfrak{p}_{\mathsf{a}j}} |\mathsf{a}, j\rangle\big)\big(\sum_k \sqrt{\mathfrak{p}_{\mathsf{a}k}} \langle\mathsf{a}, k|\big) = \sum_{jk} \sqrt{\mathfrak{p}_{\mathsf{a}j}\mathfrak{p}_{\mathsf{a}k}} |\mathsf{a}, j\rangle\langle\mathsf{a}, k|. \tag{4.5}$$

Density matrices introduce a more comprehensive description of quantum states than state-vectors. They naturally describe mixed as well as pure states.

Notational Interlude

I use several forms of notation in this book. The agent-states introduced above are unit-trace density matrices symbolized as $\mathsf{q}, \mathsf{p}, \mathsf{s}$, with the canonical coordinate pair q, p playing central roles. Their bold-face indicates they are vector or tensor objects (rather than scalars, which are plain-face). For the most part I do not decorate vectors and density matrices in Dirac "bras and kets." For example, the *jth* state of the position agent as a projector (density matrix) in Dirac notation is

$$|\mathsf{q}, j\rangle\langle\mathsf{q}, j|$$

which, whenever possible, I compactly express as

$$\mathsf{q}_j \equiv |\mathsf{q}, j\rangle\langle\mathsf{q}, j|.$$

I understand $\mathsf{q}(t), \mathsf{p}(t)$ as a dynamical pair of coordinates for a single degree of freedom extending the Theory of Classical Mechanics: they evolve as density matrices in time t under the laws of motion to become whatever they will given initial conditions. The time dependence will not be explicit for much of the presentation, only appearing when time-evolution is the focus.

Agent-states, which are density matrices, are to be distinguished from agent-state-*vectors*. Agent-state-vectors are the eigenstates (eigenvectors) of these density matrices. Each agent-state possesses a set of eigenvectors and their Hermitian duals. I take no notational shortcuts with state-vectors but fully array them in their Dirac garb,

$$|\mathsf{q}, j\rangle, \quad \langle\mathsf{q}, j| \qquad |\mathsf{p}, j\rangle, \quad \langle\mathsf{p}, j| \qquad j = 1, 2, \ldots, d.$$

An agent-state's eigenvectors also constitute the set of agent projectors, (the self-outer product of the state-vector). These are individual unit-trace density matrices for each agent, I shall sometimes abbreviate (as above) as

$$\mathsf{q}_j \equiv |\mathsf{q}, j\rangle\langle\mathsf{q}, j|, \qquad \mathsf{p}_j \equiv |\mathsf{p}, j\rangle\langle\mathsf{p}, j|.$$

A general agent-state, such as a, has a state-vector which can evolve into mixed states (a probabilistic mixture of pure states), will be indicated as

$$\mathsf{a} = |\mathsf{a}\rangle\langle\mathsf{a}|$$

when it is in a pure state (one of its eigenvectors).

Given a state-vector: an agent-state density matrix is immediately implied as its projector and is a pure state. The map is one-to-one. The inverse process, given an agent-state density matrix: a unique pure state is not implied. This is because a density matrix possesses an ensemble of eigenvectors: the map is one-to-many. Such is the richer world of density matrices—and the quantum world they describe.

Fundamental Representation

A complete set of eigenvectors for a quantum state a compose a set of generalized projectors. In the Theory of Autonomous Quanta, these are the quantum states available to the agent a, the set of d^2 possible states, the matrix of states or *state matrix* of the agent a:

$$|\mathsf{a}, j\rangle\langle\mathsf{a}, k|, \qquad j, k = 1, 2, \ldots, d. \tag{4.6}$$

The quantum states $|\mathsf{a}, j\rangle\langle\mathsf{a}, k|$ can also be understood as labelled "containers" open to receiving content. The content of the container is the probability the agent is in the state identified by the container. The agent probability distribution $\mathfrak{p}_{\mathsf{a}j}$, $j = 1, 2, \ldots d$ contains the agent's essential information in its correlations,

$$\mathfrak{p}_{\mathsf{a}j}, \qquad \mathfrak{c}_{\mathsf{a}jk} = \sqrt{\mathfrak{p}_{\mathsf{a}j}\mathfrak{p}_{\mathsf{a}k}}, \qquad j, k = 1, 2, \ldots, d.$$

The coefficients $\mathfrak{c}_{\mathsf{a}jk} \equiv \sqrt{\mathfrak{p}_{\mathsf{a}j}\mathfrak{p}_{\mathsf{a}k}}$ constitute a *coherence* matrix of the quantum correlations of the agent. The entries on the diagonal are"self-correlations." They constitute a probability vector for the states' existence. The state of a quantum agent is the projection of its coherence matrix on its state matrix to form the density matrix of the agent (4.5),

$$\mathsf{a} = |\mathsf{a}\rangle\langle\mathsf{a}| = \sum_{jk} \mathfrak{c}_{\mathsf{a}jk}|\mathsf{a}, j\rangle\langle\mathsf{a}, k|. \tag{4.7}$$

I call this representation in terms of the coherence matrix of the agent the *fundamental representation.* It is composed of the projections of the the agent's coherence matrix $\mathfrak{c}_{\mathsf{a}}$ on the agent's state matrices $|\mathsf{a}, j\rangle\langle\mathsf{a}, k|$ spelled out in Eq.(4.7) above.

Canonical Representation

It is possible to distinguish classical behavior from quantum by separating out the diagonal and skew parts of the coherence matrix.

Principle (Canonical Representation)**.** *The support of a complete agent-state* (4.7) *may be divided into a sum of mixed states with classical probabilities (the first term) and quantum correlations (the second term):*

$$\mathsf{a} = \sum_{j} |\mathsf{a}, j\rangle\mathfrak{p}_{\mathsf{a},j}\langle\mathsf{a}, j| + \sum_{j\neq k} \mathfrak{c}_{\mathsf{a}jk}|\mathsf{a}, j\rangle\langle\mathsf{a}, k|. \tag{4.8}$$

Since the probability is a number, it can be moved across the factors of the projector and "dropped in between the container walls" as content as was done above,

$$\mathfrak{p}_j|\mathsf{a}, j\rangle\langle\mathsf{a}, j| \to |\mathsf{a}, j\rangle\mathfrak{p}_{\mathsf{a}j}\langle\mathsf{a}, j|,$$

a useful mnemonic that captures the address as a "container" in Hilbert space whose content is the probability the addressed state is occupied.

Projectors $\mathsf{a}_j \equiv |\mathsf{a}, j\rangle\langle\mathsf{a}, j|$ *are the unit-trace density matrices of individual states. Apportioned by probabilities, their sum is the "classical" portion of the complete agent-state* (4.8) *appearing in the first term.*¶ *The coherences* $\mathfrak{c}_{\mathsf{a}jk} \equiv \sqrt{\mathfrak{p}_{\mathsf{a}j}\mathfrak{p}_{\mathsf{a}k}}$, $j \neq k$ *identify the second term of correlations* $|\mathsf{a}, j\rangle\langle\mathsf{a}, k|$. *They contain the "quantum" portion of a complete agent-state.*

¶A projector of a superposition of pure states, as in Eq. (4.8) is not equal to the superposition of a sum of pure state projectors as in the first term. The difference is in quantum correlations.

I shall refer to this representation (4.8) of agent-states as *canonical.* The canonical representation is exact, that is, it is a re-arranged version of the exact quantum density matrix (4.7).

Quasi-Quantum Representation

The classical world is evidenced in overwhelming (but not all) appearances of nature for which quantum states are typically mixed with probabilities $\mathfrak{p}_{\mathsf{a}j}$ and quantum coherences $\mathfrak{c}_{\mathsf{a}jk}$, $j \neq k$ have been decohered by agent interactions. [13].

Throughout most of this book I adopt a *quasi-quantum representation* at the intersection of the classical and quantum rather than the exact canonical representation. I propose that quantum agents as well as human agents adopt such a stance in the face of quantum nature. In this perspective, states are generally mixed, respecting their coherent quantum origin and coherences generally (but not always) vanish, respecting their classical decoherence at the hand of agent observations/interactions. At the limit in which only probabilities survive in the description of a quantum system, an agent state has the quasi-quantum representation,c

$$\mathsf{a} = \sum_j \mathfrak{p}_{\mathsf{a}j}\mathsf{a}_j = \sum_j |\mathsf{a}, j\rangle \mathfrak{p}_{\mathsf{a},j} \langle \mathsf{a}, j|. \tag{4.9}$$

The Question of Basis

In human-centric quantum mechanics, human agents establish a basis for their theoretical and experimental activity and work in *that basis.* In quantum computing, one works frequently in the "computational basis." In Autonomous Quantum Theory the states (4.6) are understood to arise naturally in the dynamics of matter independently, for the most part, from human agents. The states *are the basis.* It is in this spirit that I say "autonomous quanta carry their own basis."

The Qubit Automaton

Definition (Qubit Automaton)**.** *The qubit automaton is a mathematical machine composed of four interacting sub-machines or sub-automata. Three of these machines are software representatives of the physical force field, dynamically mapping their states-of-being into one another. The fourth, the identity* I *is a logical machine that maps any state to the state itself and is invariant.*

The qubit automaton is an elementary analog of and homolog to more complex compartmentalized structures in nature containing sub-parts, such as entangled molecules, molecular capsules and cages, gel and film enclosures, and biological vesicles containing vesicles within vesicles. The qubit automaton is the software representation of these structures at the bottom end of the hierarchy—the elementary computational particle.

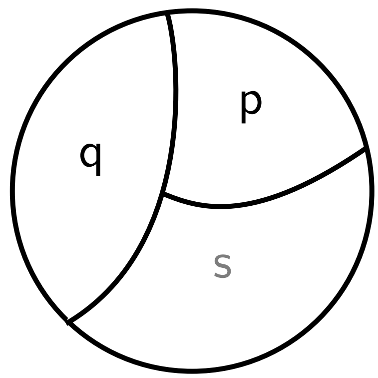

Figure A. The Qubit Automaton. Three physical agents $\mathsf{q}, \mathsf{p}, \mathsf{s}$ constitute the qubit automaton. Two are independent. In this rendering the state of the automaton is fixed by the position and momentum agents q, p. The third determined by them is the action $\mathsf{s} = i[\mathsf{q}, \mathsf{p}] + \frac{1}{2}\mathsf{I}$ and is greyed out to show it is not independent.

Principle (Information Content: 2 bits.)**.** *It takes 2 bits of information to fix the configuration of a qubit automaton because it is composed of two independent operators, each an agent that can act as well as hold a state of 1 bit. Such a wider repertoire of behavior of the qubit automaton requires two independent bits of information rather than the single bit which suffices for the "helpless" qubit in its vector subspace.*

As noted previously, a full-rank qubit automaton is specified by 3 real numbers. These include two real numbers to specify the single component of coherence and one real number to specify the normalized probability distribution on the diagonal. Complex systems with many degrees of freedom appear in the computational representation as systems of qubit automata, a tensor product network of automata described in Chapter 8.

Automaton Memory Registers and their Content

The existence of computational registers within the structure of quantum operator algebra is striking. Quantum automata possess memory registers that hold their state of being. The address of the register is the agent's state in Hilbert-Schmidt space. The content of the register is an occupation probability for the state. Addresses are eigenstates. Contents are eigenvalues (magnitudes).

Principle (Agent Memory Registers)**.** *Let* a *represent any instance of the three physical agent-states* $\mathsf{q}, \mathsf{p}, \mathsf{s}$ *with eigenvalues* a_j *and eigenstates* $|\mathsf{a}, j\rangle$. *An agent's eigenstates in the form of its projectors are natural memory registers,*

$$\mathsf{a}_j = |\mathsf{a}, j\rangle\langle\mathsf{a}, j|, \qquad \sum_j \mathsf{a}_j = \mathsf{I}, \quad j = 1, 2.$$

Each register represents a state of the agent. Projectors as memory registers are rank-1 matrices formed as outer products of eigenvectors $|\mathsf{a}, j\rangle\langle\mathsf{a}, j|$ *identifying states as containers open to receiving content. The set of registers is assumed complete and sum to the identity.*

On the classical level, only one register can be excited by the dynamics at any time. This is consistent with classical understanding that a dynamical variable—a canonical coordinate—cannot be in more than one state at a time and it must appear in that definite state someplace in the state space. Because a singleton appearance in the state space is not required on the quantum level, according to the superposition principle, a probability distribution over states emerges at a foundational level. A normalized probability implies that probabilities sum to unity and that the trace of an agent-state must likewise be unity.

Principle (Quantum Probability)**.** *The content of an automaton register is the probability that the register* a_j *holds the state of agent* a. *This probability is proportional to the magnitude of the agent-state's eigenvalues* $\sqrt{\mathfrak{p}_{\mathsf{a}j}}$. *The excitation of an agent register is described by a probability* $\mathfrak{p}_{\mathsf{a}j}$ *which is the probability the* j*th register of agent* a *is excited. An agent-state is the sum of its registers excited with their appropriate probabilities has the quasi-quantum form*

$$\mathsf{a} = \sum_j |\mathsf{a}, j\rangle \mathfrak{p}_{\mathsf{a}j} \langle\mathsf{a}, j|, \qquad \sum_j \mathfrak{p}_{\mathsf{a}j} = 1, \quad Tr(\mathsf{a}) = 1, \qquad j = 1, 2. \tag{4.10}$$

Probabilities—which are the key carrier of the agent's state—emerge naturally from agent operators: they are the magnitudes of its eigenvalues. The probability $\mathfrak{p}_{\mathsf{a}_j}$ is to be read as "the probability the agent a is in state a_j." As the agent-state is a unit trace density matrix, the eigenvalues sum to unity, as must the probabilities.

Vanishing probabilities describe empty registers. An agent that *statistically* contains no information has all probabilities uniform as $\mathfrak{p}_{\mathsf{a},j} = 1/d$. Since no register is distinguished, their sum collapses to the normed identity —an agent possessing a completely random dynamic and indicated by a random statistical state,

$$\mathsf{a} = \sum_j |\mathsf{a}, j\rangle \frac{1}{d} \langle \mathsf{a}, j| = \frac{1}{d}\mathsf{I} = \mathbf{i}.$$

The qualification 'statistical' is important. I understand agents receiving a definite signal on each trial. If the agent is in an indefinite state, the observation is not repeatable under otherwise fixed conditions. Indefinite observations result in random behavior and are described in the theory by the random movement of the observing agent (Chapter 4, 11).

The trials are repeated in nature on a frequency governed by the energy H of the force field interaction, $\omega \approx H/\hbar$. The probabilities are a property of the statistical nature of the theory. They should be understood as a matter of principle as an average over many trials under fixed conditions most controlled in a laboratory setting.

> Quantum agent-states (4.10) are hereby demonstrated as natural computer memories. Registers of the memory identify states in Hilbert-Schmidt space. Register contents are the agent's eigenvalue magnitudes (probabilities) which indicate whether or not the state is occupied. The occupation can be singleton for a definite state or a probability distribution for general mixed states.

The state of the qubit automaton is fixed by two independent agents with their attendant memories and probabilities which I typically call the 'position' and 'momentum' agents.

The most general agent-state of the qubit can be represented as

$$\boldsymbol{g}(\mathfrak{p}, \mathfrak{c}) = \begin{pmatrix} \mathfrak{p} & \mathfrak{c} \\ \mathfrak{c}^* & 1 - \mathfrak{p} \end{pmatrix}.$$

Each agent-state is described by 2 parameters: a probability $\mathfrak{p}$ and a coherence $\mathfrak{c}$. The probability can be taken as a real number for the agent to be in the first of its two diagonal states, $\mathfrak{p}_1 = \mathfrak{p}$ (for which the probability for the second is $\mathfrak{p}_2 = 1 - \mathfrak{p}$). The coherence $\mathfrak{c}$ is a complex number that specifies both off-diagonal components because the agent-state is Hermitian: $\mathfrak{c}_{21} = \mathfrak{c}_{12}^*$. Since the coherence is complex, the qubit automaton is described by 3 real numbers.

Illustration: The Pauli States

Quantum agents can be illustrated for the simplest Hermitian representation, the Pauli States. Pauli states have values $\boldsymbol{g}(\mathfrak{p}, \mathfrak{c})$:

$$\hat{\mathsf{q}} = \boldsymbol{g}(1,0), \qquad \hat{\mathsf{p}} = \boldsymbol{g}(1/2,1/2), \qquad \hat{\mathsf{s}} = \boldsymbol{g}(1/2,i/2), \qquad \mathsf{I} = \boldsymbol{g}(1/2,1/2)$$

$$\hat{\mathsf{q}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{\mathsf{p}} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \hat{\mathsf{s}} = \frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}, \quad \mathsf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{4.11}$$

The corresponding agent-actors are

$$\hat{\mathsf{Q}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \hat{\mathsf{P}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\mathsf{S}} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \mathsf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{4.12}$$

Simple and instructive as the Pauli states can be, quantum agents in motion will only be in Pauli states under exceptional conditions. Agents in generic motion can be expected to be far more various with entries over the complex number field subject to normalization.

Agent-states and actors have eigenstates in Hilbert space sub-spaces of Hilbert-Schmidt space symbolized as $|\mathsf{q}, j\rangle, |\mathsf{p}, j\rangle, |\mathsf{s}, j\rangle$ where the first entry $(\mathsf{q}, \mathsf{p}, \mathsf{s})$ identifies the agent and the second entry j identifies the agent's eigenstate (and projector). Here they are in the Pauli state:

Position

$$|\hat{\mathsf{q}}, 1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |\hat{\mathsf{q}}, 2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{4.13}$$

Momentum

$$|\hat{\mathsf{p}}, 1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad |\hat{\mathsf{p}}, 2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \tag{4.14}$$

Action

$$|\hat{\mathsf{s}}, 1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix}, \quad |\hat{\mathsf{s}}, 2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}. \tag{4.15}$$

A set of six rank-1 projectors (two for each of the three physical agents) in Pauli state is populated by these eigenstates:

$$\hat{\mathsf{q}}_1 = |\hat{\mathsf{q}}, 1\rangle\langle\hat{\mathsf{q}}, 1| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \hat{\mathsf{q}}_2 = |\hat{\mathsf{q}}, 2\rangle\langle\hat{\mathsf{q}}, 2| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{4.16}$$

$$\hat{\mathsf{p}}_1 = |\hat{\mathsf{p}}, 1\rangle\langle\hat{\mathsf{p}}, 1| = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \qquad \hat{\mathsf{p}}_2 = |\hat{\mathsf{p}}, 2\rangle\langle\hat{\mathsf{p}}, 2| = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \tag{4.17}$$

$$\hat{\mathsf{s}}_1 = |\hat{\mathsf{s}}, 1\rangle\langle\hat{\mathsf{s}}, 1| = \frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}, \qquad \mathsf{s}_2 = |\hat{\mathsf{s}}, 2\rangle\langle\hat{\mathsf{s}}, 2| = \frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}. \tag{4.18}$$

You have before you the internal structure of the qubit automaton in the Pauli state. Each of these is a pure state in Hilbert-Schmidt space. The position states (4.16) are clearly so. The momentum and action states (4.17)-(4.18) are readily checked to each satisfy the purity condition: the trace of their squares is unity.

Observe that these agent-states are pure but they are not all diagonal. Indeed, they cannot be because they have to represent three agents (beyond the logical agent, the identity) and only two diagonal operators exist in the qubit automaton's set of four. Observe also that these matrices are highly degenerate, as they should be: they describe pure singleton subspaces.

Canonical Representation of Pauli States

A skew Pauli state, such as $\hat{\mathsf{s}}_1$, provides an illustration of the fundamental and canonical representations, (4.7)-(4.8). The representation

$$\hat{\mathsf{s}}_1 = \frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$$

is fundamental with coherence matrix,

$$\mathfrak{c}_{\hat{\mathsf{s}}_1} = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

Its canonical representation is

$$\hat{\mathsf{s}}_1 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} 0 & -\frac{i}{2} \\ \frac{i}{2} & 0 \end{pmatrix},$$

with probability vector

$$\mathfrak{p}_{\hat{\mathsf{s}}_1} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix},$$

and coherence matrix,

$$\mathfrak{c}_{\hat{\mathsf{s}}_1} = \begin{pmatrix} 0 & -\frac{i}{2} \\ \frac{i}{2} & 0 \end{pmatrix}.$$

The Random Agent-State and its Null Agent-Actor

Uncertain events and their probabilities inject a statistical ingredient into the theory of quantum automata. The *random* manifests itself in quantum agency in the fully random agent-state $\mathbf{i} = \frac{1}{d}\mathsf{I}$, the normed identity, and its agent-actor, the null-operator $\mathbf{0}$ (whose matrix representation entries consist of zeros);

$$\mathbf{i} = \frac{1}{d}(\mathsf{I} + \mathbf{0}), \quad \mathbf{0} = \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}. \tag{4.19}$$

The random agent-state represents the opposite extreme to the fully definite agent: the singleton state. The random agent-state's actor, the null operator, maps each agent to the null operator itself and is the counterpart to the identity which maps each agent to the agent itself.

The random agent-state $\mathbf{i}$ holds "all states" as possibilities and will play a key role in concert with the canonical agents q, p, particularly in tensor product automata (Chapter 8, 10).

Probability: Definite and Indefinite in Autonomous Quantum Theory

A theory describes not an individual instance of some phenomenon in nature but rather a statistical average of many instances. On the other hand, a statistical theory can reproduce definite results when such results are permitted—as they indeed can be when the probability is singleton. A definite state is repeatedly produced as output under fixed experimental inputs. An indefinite, or mixed, state has more than one non-zero eigenvalue and is therefore partly random. it cannot be repeatedly produced from the same fixed inputs but fluctuates between its non-zero eigenvalues in proportion to their probabilities. A fully random state has all eigenvalues equal.[‖]

The theory of quantum automata occupies a "middle ground" between assertions about instantaneous physical events and statistical descriptions over an ensemble of such events. Instantaneous superposed states are described by complex square-root probability amplitudes and coherences. Superposed states are decohered under observation to mixed states wherein square-root probability amplitudes become real-valued probabilities (4.9). The middle ground exists because some physical situations produce definite observations even though most do not. The situation is summarized in this principle,

Principle (The Probable contains the Definite as Singleton)**.** *A definite description of matter always exists within a statistical description at an abstract mathematical*

[‖]It is a fact that confirmation of theory of specific behaviors (such as the spins of an electron in the Stern-Gerlach experiment) requires multiple trials for those observations that are not definite. Such multiple trials under fixed conditions are done in a laboratory or laboratory-like setting.

level. A definite description is that for which its probability distribution is singleton. The Heisenberg Principle (Chapter 6) declares that not all agents can be observed with singleton probability distributions. Quantum matter therefore cannot be fully in definite states where all independent agents hold definite values.

I have not notationally distinguished statistical quantities in the theory from "instantaneous" or "single-trial" values. They are distinguished by the foregoing principal. One can summarize by declaring all agent-states statistical with the definite states revealing themselves as singleton states when the situation permits.

From Probabilities to Physical Values

Agent-states are composed of a superposition of eigenstates and their probability amplitudes. They do not become actual until they are observed and the probability amplitudes show themselves as statistical quantities issuing from the observation. These are clearly theoretical constructs which can only be experimentally rendered in controlled experiments, the best theory can do with matter that possesses intrinsic uncertainty.

Agent states have physical values as positions, momenta, energy, and other quantities. Let q_o, p_o be the characteristic physical values of a quantum agent, for example, the ground state radius q_o and orbital momentum p_o of the hydrogen atom satisfying.

$$p_o q_o = \hbar$$

where the Planck constant $\hbar$ appears.

Physically valued agent-states (position, momentum, ...) corresponding to their non-dimensional agent-states q, p are symbolized with an overbar as

$$\bar{\mathsf{q}} = q_o \mathsf{q}, \qquad \bar{\mathsf{p}} = p_o \mathsf{p},$$

and may be dimensional, as in meters, kg-meters/sec, etc. The eigenvalues of these dimensional agent-states are dimensional scalar observables, the objects to be manifest in physical situations, such as measurements in experiments or encountered in nature. The product of physical values of states and their probabilities gives the expected physical value or the average value of the set of measurements of that physical state.

The Question of Time

The ensemble average of an observed quantity $(\mathsf{h}(t) \parallel \mathsf{a}(t))$ is understood as an average over a small window in time centered on the time t.

Canonical Unitary Transformations

Coordinate language that describes a quantum system is not unique. The description in coordinates q, p with Hamiltonian $\mathsf{h}(\mathsf{q}, \mathsf{p})$ is physically equivalent to that in language q', p' with Hamiltonian $\mathsf{h}'(\mathsf{q}', \mathsf{p}')$ where the two languages are related by a canonical unitary transformation U:

$$\mathsf{q}' = \mathsf{U}^\dagger \mathsf{q} \mathsf{U}, \qquad \mathsf{p}' = \mathsf{U}^\dagger \mathsf{p} \mathsf{U}.$$

Unitary transformations maintain the invariance of the inner product of state-vectors and operators. A unitary transformation matrix satisfies

$$\mathsf{U}^\dagger \mathsf{U} = \mathsf{I}.$$

The Hamiltonian is invariant in both languages

$$\mathsf{H}(\mathsf{q}, \mathsf{p}) = \mathsf{H}'(\mathsf{q}', \mathsf{p}')$$

with

$$\mathsf{h} = \frac{1}{2}[\mathsf{I} + \mathsf{H}(\mathsf{q}, \mathsf{p}) = \mathsf{h}' = \frac{1}{2}[\mathsf{I} + \mathsf{H}'(\mathsf{q}', \mathsf{p}').$$

The canonical character of the transformation implies the equations of motion are the same in both languages,

$$\frac{d\mathsf{q}'}{dt} = (i/\hbar)[\mathsf{q}', \mathsf{H}'(\mathsf{q}', \mathsf{p}'], \quad \frac{d\mathsf{p}'}{dt} = (i/\hbar)[\mathsf{p}', \mathsf{H}'(\mathsf{q}', \mathsf{p}'].$$

Since the Identity makes no contribution to the commutator, equations of motion are the same within a factor of 2 whether the Hamiltonian appears as agent-actor H or as agent-state h:

$$\frac{d\mathsf{q}'}{dt} = (2i/\hbar)[\mathsf{q}', \mathbf{h}'(\mathsf{q}', \mathsf{p}'], \quad \frac{d\mathsf{p}'}{dt} = (2i/\hbar)[\mathsf{p}', \mathbf{h}'(\mathsf{q}', \mathsf{p}'].$$

Species and Varieties of Agents

Principle (Two Independent Agents for Each Degree of Freedom)**.** *Each elementary degree of freedom in nature corresponds to a qubit automaton with a pair of independent agents—a canonical position and momentum. A system with N freedoms possesses $d = 2N$ such independent canonical agents just as in a classical mechanical system.*[**]

[**]This is true even though the total number of agents—Hilbert-Schmidt operators—is $d^2 - 1 = 4N^2 - 1$ in number. The total number of operators are not independent because they are linked by unitary algebraic dependence (4.2), (4.3). With dependency accounted for there are $d = 2N$ independent freedoms.

The number of agents that have a diagonal representation is $(d-1)$ (in addition to the identity). These agents commute. This leaves $(d^2-1)/(d-1) = d+1$ agents that do not. Quantum agents therefore fall into two sets: a set of $d-1$ agents that commute and share information and $d+1$ sets each containing $d-1$ agents that do not. Agents for which information exchange is indefinite (observations give random results) are said to be complementary following Bohr.

Let each of these $d+1$ sets define a quantum species: within a species information exchanged in interactions is definite, but agents of different species cannot exchange definite information. These commuting sets are also described as quantum contexts in the literature.[20] The $d-1$ agents within a species are the varieties of the species: varieties interact exchanging definite information.††

Principle (The Qubit Automaton Contains 3 Species of 1 Variety Each.). *The qubit automaton ($d = 2$) has $d^2 - 1 = 3$ species, each species possessing 1 variety.*

The two independent physical agents of the qubit may be deployed as one diagonal operator and one skew-Hermitian operator. The diagonal operator manifests the 'classical' correlations of a quantum system in that a set of diagonal operators is equivalent to a set of numbers satisfying ordinary rather than matrix algebra. The skew operator bears the quantum correlations. In the Pauli state, this assignment is held by the momentum agent while the position agent holds the 'classical' diagonal representation. The terms "position" and "momentum" are name tags for the canonical agent pair. They can be but need not be a literal mechanical position and momentum.

Quantum species are fluid and can change in dynamic interaction. The representation an agent holds will shift in time-evolution with the balance between the 'classical' and 'quantum' character of the agent correspondingly evolving depending upon the particular force-field and Hamiltonian governing the system (agents become 'more diagonal or less diagonal'). [2]

Superposition and Context

The superposition principle is a hallmark of quantum mechanics. Quantum species and their contexts are equally significant aspects of quantum mechanics. The two are related by linearity as I shall now demonstrate using the canonical agents, position and momentum in Pauli states.

††As an analogy with biological species, when agents of the same species interact—mate—they produce a definite offspring of the mating varieties. When agents of different species interact, they produce no offspring. Sharing definite information is the counterpart to the ability to mate. It is intrinsic to the quantum information space that there will always be an information deficit between the total number of operators and diagonal operators that commute.

Equal superpositions of Pauli position agents (sum and difference)

$$\frac{1}{\sqrt{2}}(|\hat{\mathsf{q}},1\rangle + |\hat{\mathsf{q}},2\rangle), \qquad \frac{1}{\sqrt{2}}(|\hat{\mathsf{q}},1\rangle - |\hat{\mathsf{q}},2\rangle) \tag{4.20}$$

turn out to be the Pauli momentum agents, (as can be checked from the results above)

$$\frac{1}{\sqrt{2}}(|\hat{\mathsf{q}},1\rangle + |\hat{\mathsf{q}},2\rangle) \equiv |\hat{\mathsf{p}},1\rangle, \qquad \frac{1}{\sqrt{2}}\hat{(}|\mathsf{q},1\rangle - |\hat{\mathsf{q}},2\rangle) \equiv |\hat{\mathsf{p}},2\rangle. \tag{4.21}$$

This result can be formulated in a principle which emphasizes the power of superposition to change contexts. Superposition is a unitary transformation:

Principle (Superposition and Complements)**.** *An equal superposition of the agent-states of a given species is the complementary agent-state.*

Agent Environments as Open Quantum Systems

Observing agents form an environment for the agent observed. In this perspective, one can regard each agent as existing in an environment of other agents. Framed in this way, the well-developed theory of quantum operations and decoherence of open quantum systems [12, 13] is directly applicable to a closed system of quantum automata according to the following principle:

Principle (Quantum Agent Environments)**.** *Every quantum agent exists in an environment formed by all other agents. An individual quantum agent may therefore be regarded as an open quantum system residing in the environment of all other agents*

The principle of quantum agent environments makes the extensive theory of Open Quantum Systems complementary to Automata Theory [13, 12] but with a limiting caveat. Theories of quantum operations are constrained in their predictive power because the dynamics they describe are subject to the spontaneous appearance of unpredictable states. *Quantum Operations* acquire their theoretical utility in the hands of conscious agents who structure them in coherent experimental configurations.

Open Quantum Systems reach their most natural capacities of description in describing mixing, relaxation, and other decohering operations *on quantum systems that have been ordered by processes (including the actions of conscious agents) that are beyond their capability.* Open Quantum Systems Theory has little if any capacity to describe genuinely creative dynamics in which order and structure come into being from a random sea of possibility.

Summary

"Position-momentum" operator coordinates of a quantum agent

$$\mathsf{q}_j = |\mathsf{q}, j\rangle\langle\mathsf{q}, k|, \qquad \mathsf{p}_j|\mathsf{p}, j\rangle\langle\mathsf{p}, k|,$$

can be regarded as the addresses of computational registers whose contents are coherences: "square-root" correlations between the occupation probabilities $\mathfrak{p}_{\mathsf{q}j}, \mathfrak{p}_{\mathsf{p}k}$ for each pair of registers j, k,

$$\mathfrak{c}_{\mathsf{q}jk} = \sqrt{\mathfrak{p}_{\mathsf{q}j}\mathfrak{p}_{\mathsf{q}k}}, \qquad \mathfrak{c}_{\mathsf{p}jk} = \sqrt{\mathfrak{p}_{\mathsf{p}j}\mathfrak{p}_{\mathsf{p}k}}.$$

The smallest such system is that for a single degree of freedom, the qubit automaton, for $j, k = 1, 2$.

The qubit automaton is the software representation of the most elementary physics in nature: a single quantum degree of freedom ranging from a single electron spin, a two-level atom, and beyond. An isolated degree of freedom—an isolated qubit shielded from interaction with the rest of the universe —is a highly idealized object like an idealized, isolated atom. Nonetheless it is not without structure.

The qubit automaton contains two independent agents interacting with one another according to the canonical position-momentum paradigm of mechanics. The independent agents are quantum machines each with two machine states. This interaction synthesizes both observation and motion—change of state—as will be described in the following chapters.

A qubit holds 1 bit of definite information. The qubit automaton holds two bits of information, but only 1 bit can be definitely observed. That means one can query it for 1 bit of information and receive a definitive answer as to whether it is in the off-state or on-state. When one more properly queries the full population of the qubit automaton one will be querying both agents. Though these agents hold two bits of independent information, a query will yield at most 1 bit of definite information. The remaining bit will be presented randomly.

Chapter 5

Matter-Information Duality: Agent Eigenstates and Eigenvalues

Quantum mechanics entered the physics world at the beginning of the twentieth century with the arrival of the "wave function." The Theory of Mechanics (destined to become the Theory of *Classical* Mechanics) was well-perfected in canonical position-momentum coordinates under the direction of a Hamiltonian. Why, then, the wave function?

The wave function did what classical particle mechanics could not do: make sense of the particulate absorption and emission of light and discrete energy levels exhibited by atoms. But where did this new kid on the block fit in the well-established structure of Canonical Mechanics? What did the wave function have to do with least action, the Hamiltonian, and its canonical coordinates?

The wave function was all about information, its 'waves,' amplitudes of probability. Information had been recognized in the domain of matter and energy to become an equal partner. The quantum would now be seen to be as much about information in motion as about "microscopic particles in motion."

I shall show that the canonical structure of mechanics continues into in the quantum world under a mechanical and mathematical transfiguration: the functions of classical canonical coordinates are lifted from point functions to become operators (Chapter XXXX.) Quantum operators possess matter-information duality.

The matter-information duality of quantum agents is prominently exhibited in their operator structure. Operators are composed of an interlocking system of two parts: eigenstates and eigenvalues. Eigenstates embody the material structure of a quantum, the physical structure of the force field in shaping matter, the lay of energy fields in space and time, the architecture of chemical bonds.

The physical world would henceforth be known as a matter-information or energy-information duality (with matter understood as mass-energy).

Principle (Mass-Energy Structure of Matter Resides in Eigenstates)**.** *The matter face of a quantum agent* a *is contained in its eigenstates.*

Different eigenstates have different physical structure. For example, the physical structure of the hydrogen atom is embodied in the eigenstates shown in Figure 5.X. Each structure constitutes a quantum state. This is the mass-energy face of the hydrogen atom.

The information face of a quantum system is manifest in selection, in choice of eigenstate. in which of the eigenstate structures does a quantum agent take form? The information is contained in the probability amplitude $\mathfrak{p}_{\mathsf{a}j}$ which gives the probability agent a is in the eigenstate a_j:

$$\mathfrak{p}_{\mathsf{a},j} : \quad \mathsf{a} \to \mathsf{a}_j = |\mathsf{a}, j\rangle\langle\mathsf{a}, j|.$$

Principle (Information of Matter Resides in Eigenvalues)**.** *The information face of a quantum system is contained in its eigenvalues.*

The superposition principle in concert with the linearity of quantum mechanics declares the agent's state a sum over the possible eigenstates multiplied by their probabilities, here in the quasi-quantum representation (4.9):

Principle (Unity of the Duality of Matter and Information)**.** *A quantum agent-state*

$$\mathsf{a} = \sum_j \mathfrak{p}_{\mathsf{a}j}\mathsf{a}_j = \sum_j |\mathsf{a}, j\rangle\mathfrak{p}_{\mathsf{a},j}\langle\mathsf{a}, j|. \tag{5.1}$$

is composed of eigenstates $\mathsf{a}_j = |\mathsf{a}, j\rangle\langle\mathsf{a}, j|$ *(physical structures) and the probabilities* $\mathfrak{p}_{\mathsf{a}j}$ *(quantum information) that the agent exhibits that physical structure. Quantum agent-states are the outstanding example of the duality of matter and information in the natural world.*

Chapter 6

Energy and Information: The Hamiltonian Superagent

Qubit automata in action manifest the dynamism of nature: a modern scientific understanding of the duality of Aristotelian *matter and form.* The duality appears as *mass-energy* on the one hand and and *quantum information* on the other, as "hardware" in particle force fields and as "software" in a *Hamiltonian or energy operator** that directs hardware motion. As the global energy agent, the Hamiltonian superposes the energy contributions of each of the canonical agents into a whole that *moves* hardware. As the global information agent, the Hamiltonian combines information contributions from each agent that, *directs* hardware. Because of its overarching role in the dynamic of motion, I shall call the Hamiltonian a *superagent.*

> The cast of agents at the foundation of nature are canonical position-momentum agents, a pair for each degree of freedom and a global Hamiltonian agent composed of contributions from the canonical agents over all freedoms. Canonical positions and momenta are individual agents, individuals within the collective. The Hamiltonian superagent is inherently social, a sum over contributions from each of the canonical agents. This is the same organization as that of Canonical Classical Mechanics but with coordinate functions transfigured to operators.

*This global function originates in the principle of least action from which the motion of the system flows from a single master principle: that action be an extremum.

Qubit Automaton Hamiltonians

The Hamiltonian, like the canonical position and momentum, is an agent: it has a state of being h and can also direct action as an actor H. In contrast to the canonical coordinate agents which are local and individual, the Hamiltonian is global and communal. It describes the corporate interaction of the individual agents producing output directives to the time-evolution operator to execute motion.

> The form of the Hamiltonian agent is determined by the physical configuration of the particles and their force fields. The possibilities for the qubit's energy agents are limited to superpositions of the canonical agents. Their form is dictated by the superposition of canonical coordinate eigenstates

The Hamiltonian for the isolated qubit automaton in canonical coordinates reads

$$\mathsf{H} = \mathsf{H}(\mathsf{q}, \mathsf{p}, \mathsf{s}), \qquad \mathsf{h} = \frac{1}{2}(\mathsf{I} + \mathsf{H}), \qquad \mathsf{H} = 2\mathsf{h} - \mathsf{I}.$$

The Hamiltonian agent's eigenstates are energy eigenstates. The probabilities for excitation of these eigenstates embody hold the agent's information.

Not all agents need be present in the Hamiltonian for a particular motion. The extreme in which no agents are present corresponds to a null-Hamiltonian actor (a null matrix representation) in which no actor is present in the superposition. A null-Hamiltonian directs a motion as statistical stasis: a motion frozen in time with a corresponding fully random Hamiltonian agent-state)

$$\mathsf{H} = \mathbf{0}, \qquad \mathsf{h} = \frac{1}{2}\mathsf{I}.$$

An elementary Hamiltonian for free motion has only one of the canonical agents present. The force law in play fixes the particular form of the Hamiltonian. An example is the 'free-motion' Hamiltonian

$$\mathsf{H}(t) = \mathsf{P} = const, \qquad \mathsf{h}(t) = \frac{1}{2}(\mathsf{I} + \mathsf{P}) = const.$$

This Hamiltonian describes a degree of freedom that evolves with a constant momentum traversing two states: a rotation in Hilbert space. It can be illustrated for the free motion Hamiltonian agents in the Pauli State (4.11)

$$\hat{\mathsf{h}} = \hat{\mathsf{p}} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{6.1}$$

This representation of the Hamiltonian is *fundamental* (See Eq. (4.7)).

As a single degree of freedom , the Hamiltonian agent can be in the state of either of its projectors or as a mixture of them,

$$\hat{\mathsf{h}}_1 = \hat{\mathsf{p}}_1 = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \qquad \hat{\mathsf{h}}_2 = \hat{\mathsf{p}}_2 = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{6.2}$$

In this illustration, the state is purely $\hat{\mathsf{h}} = \hat{\mathsf{h}}_1$.

A second illustrative Hamiltonian is the single qubit *"little Heisenberg"* Hamiltonian, an equal superposition of all three canonical agents,†

$$\mathsf{H} = (\mathsf{Q} + \mathsf{P} + \mathsf{S})/3, \qquad \mathsf{h} = (\mathsf{q} + \mathsf{p} + \mathsf{s})/3. \tag{6.3}$$

The Hamiltonian agent-state for the above three agents in Pauli states (4.11) in superposition is

$$\hat{\mathsf{h}} = \begin{pmatrix} \frac{2}{3} & \frac{1}{6} + \frac{i}{6} \\ \frac{1}{6} - \frac{i}{6} & \frac{1}{3} \end{pmatrix}. \tag{6.4}$$

This representation of the Hamiltonian is also fundamental

As the agents composing the Hamiltonian are in the Pauli state, the Hamiltonian is likewise in the Pauli basis. In this basis the Hamiltonian is both mixed and non-diagonal. Since this Hamiltonian state is a superposition, it is a mixed state with purity $\mathsf{Tr}\left(\hat{\mathsf{h}}^2\right) = 2/3$. Whereas the agent-states of the free-motion Hamiltonian were pure, this is not so for the little Heisenberg Hamiltonian and not so in general.

Autonomous Quantum Theory accommodates indefinite as well as definite states in a generally indefinite or statistical theory. A generally statistical description becomes definite when its probability is singleton. This happened above for the momentum agent's observation of the free motion Hamiltonian (which was itself also a momentum agent) while observations of the little Heisenberg Hamiltonian were mixed.

The two states making up the hamiltonian (6.3) in the Pauli state (6.4) are its

---

†This Hamiltonian is a single qubit Hamiltonian corresponding to its 'big brother,' the "Heisenberg Hamiltonian" for the tensor product of a pair of qubits,

$$\mathsf{H} = \frac{(\mathsf{Q}^{(1)} \otimes \mathsf{Q}^{(2)} + \mathsf{P}^{(1)} \otimes \mathsf{P}^{(2)} + \mathsf{S}^{(1)} \otimes \mathsf{S}^{(2)})}{3}, \qquad \mathsf{h} = (\mathsf{I}_{d^2} + \mathsf{H})/d^2.$$

Note that the dependent agent $\mathsf{S} = i\mathsf{PQ}$ can be eliminated so that only the canonical agents Q, P appear

$$\mathsf{Q} + \mathsf{P} + \mathsf{S} = \mathsf{Q} + 2i\mathsf{PQ} + \mathsf{P}.$$

and similarly for the agent states,

$$\mathsf{q} + \mathsf{p} + \mathsf{s} = \mathsf{q} + i[\mathsf{q}, \mathsf{p}] + \mathsf{p} + \frac{1}{2}\mathsf{I}.$$

projectors,

$$\hat{\mathsf{h}}_1 = \begin{pmatrix} \frac{1}{6}(3-\sqrt{3}) & \frac{(\frac{1}{2}+i\frac{1}{2})(\sqrt{3}-1)}{\sqrt{3}+3} \\ \frac{(\frac{1}{2}-i\frac{1}{2})(\sqrt{3}-1)}{\sqrt{3}+3} & \frac{1}{6}(3+\sqrt{3}) \end{pmatrix}, \quad \hat{\mathsf{h}}_2 = \begin{pmatrix} \frac{1}{6}(3+\sqrt{3}) & \frac{(\frac{1}{2}-i\frac{1}{2})(\sqrt{3}+1)}{\sqrt{3}+3} \\ \frac{(\frac{1}{2}+i\frac{1}{2})(\sqrt{3}-1)}{\sqrt{3}+3} & \frac{1}{6}(3-\sqrt{3}) \end{pmatrix} \tag{6.5}$$

and they are superposed with their probabilities,

$$\mathfrak{p}_{\mathsf{h}1} = \frac{1}{6}(3+\sqrt{3}), \qquad \mathfrak{p}_{\mathsf{h}2} = \frac{1}{6}(3-\sqrt{3}). \tag{6.6}$$

Though these agent-states are each tediously complex, compared to the impure Hamiltonian (6.4) their superposition creates, they are indeed pure states.

We see illustrated here the principle that an agent-state "carries its own basis." The Hamiltonian can be regarded as evolving from the interaction of canonical agents, each carrying its own basis, to the state Eq. (6.4). The corresponding dynamically evolved basis of their superposition in the Hamiltonian at this point in the evolution are the Hamiltonian's projectors, Eqs. (6.5).

Agents are Generally in Mixed Quantum States

As illustrated by the preceding examples, quantum agents are generally in mixed states [21]. This is a manifestation of the incompleteness of any theory of nature: nature cannot be fully captured in a deterministic theory.

The quantum state Eqs.(6.4)-(6.6) may also be given a *canonical* representation (Eq. (4.8)) which highlights their "classical" (diagonal) and "quantum" (skew) content. The projectors in canonical representation are divided into a "classical" portion

$$\hat{\mathsf{h}}_1 = \begin{pmatrix} \frac{1}{6}(3+\sqrt{3}) & 0 \\ 0 & 0 \end{pmatrix}, \qquad \hat{\mathsf{h}}_2 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{6}(3-\sqrt{3}) \end{pmatrix}. \tag{6.7}$$

and a "quantum" portion according to Eqs. (4.8)

$$\hat{\mathsf{h}}_1 = \begin{pmatrix} 0 & \frac{(\frac{1}{2}+i\frac{1}{2})(\sqrt{3}-1)}{\sqrt{3}+3} \\ \frac{(\frac{1}{2}-i\frac{1}{2})(\sqrt{3}-1)}{\sqrt{3}+3} & 0 \end{pmatrix}, \quad \hat{\mathsf{h}}_2 = \begin{pmatrix} 0 & \frac{(\frac{1}{2}+i\frac{1}{2})(\sqrt{3}+1)}{\sqrt{3}+3} \\ \frac{(\frac{1}{2}-i\frac{1}{2})(\sqrt{3}-1)}{\sqrt{3}+3} & 0 \end{pmatrix} \tag{6.8}$$

from which one can read off their probabilities and coherences.

Quantum agent states have density matrices that may be broadly pure and mixed, diagonal and non-diagonal. Here I single out the *purity* of a state and its *diagonality* (either naturally evolved or transformed into some diagonal basis). Diagonality is related to classicality. Purity—better, (im)purity—is related to indefiniteness and randomness. It is fundamental that a mixed (impure) state is not a single naturally evolved state but a statistical representation of uncertain situations with multiple possible outcomes.

At one end of the spectrum are pure, diagonal agent-states such as the first and last of the Pauli states—the position and the identity or fully random state (4.11). Next are non-diagonal (but pure) states such as the momentum and action, the second and third states of (4.11). Next are diagonal mixed states such as (6.7). Notice there is no unitary transformation that can make this state *both* diagonal and pure. Lastly there are states both non-diagonal and mixed, such as the state (6.4). Mixed diagonal states are clearly statistical results of observations as are all mixed states (including non-diagonal ones). Although diagonal—and therefore 'classical-like'—this is an impure state mixing the (pure) position eigenstates (4.16) in proportion to the probabilities of Eqs. (6.6). This state, which is diagonal, is in contrast to the 'quantum' state (6.4) which is a mixture of the non-diagonal pure states (6.5).

Hamiltonian Code and the Primitive Intelligence of Motion

The Hamiltonian, once established, is an invariant that directs the motion of agents. What establishes the Hamiltonian? The Hamiltonian's invariant state may be understood in two ways. In the first it is, like the question of bases and the initial state of the universe, an "act of God" outside the bounds of theory. The "Hamiltonian of the universe" is too extravagant a concept to be useful. On the other hand, nearly-isolated systems with particular features, such as a system of particles interacting with central forces or bound particles in a lattice interacting with nearest or more distant neighbors possess feasible Hamiltonians.

The Hamiltonian encodes the primitive intelligence of quantum automata. Individual canonical agents experience ("learn about") other agents in their observation of the Hamiltonian superagent —the seat of information common to the whole community of agents. The Hamiltonian embodies a *code* or program that directs dynamics. The code is constructed from the actions available to individual degrees of freedom—qubit automata—according to the elementary force laws and sustained by those force fields..

I propose that this code, embodied in the Hamiltonian agent's registers, is the object of individual agent observations as described in the following chapter. Agent's execute their observed Hamiltonian code in a time-evolution operator to direct their motion. The simple intelligence of the automaton is to (1) observe the Hamiltonian code informed by the current state of the canonical agents, (2) compute (move) agents according to that code to new states, and (3) repeat that process in the next moment in time. The primitive sequence of the agent action is

$$\textit{observe-compute-move-repeat-} \cdots$$

and will be described in detail in the following chapters.

One can recognize a primitive, elementary process in mechanics that re-appears as a pattern in higher emergent levels, most notable, in life for which amino acids, DNA code, and ribosomes produce proteins. Amino acids are analogues to individual agent-states, Hamiltonian code to DNA code, and ribosomes to the time-evolution operator. In this sense, the Hamiltonian code is a forerunner and primitive basis for the operation of all natural codes, such as the organic codes [26].

Summary

Quantum automata are composed of individual canonical coordinate agents and a global superagent—the Hamiltonian—that holds the information of the community of individual agents.

Chapter 7

Quantum Observation

The importance of observation—information exchange between agents—as fundamental to physical reality was driven home by J. A. Wheeler:

Principle (Wheeler Observation Principle)**.** *No phenomenon is a physical phenomenon until it is an observed phenomenon. —J. A. Wheeler**

Wheeler's Principle that "a phenomenon is not a physical phenomenon until it is an observed phenomenon" is a difficult principe to abide if one regards observers as human beings in a macroscopic world. But If observers are nanophysical agents activated by the force field whose observations are more properly understood as received signals from other nanophysical agents, a compelling picture emerges. This process, without the recognition of agency, is a familiar and well-accepted behavior in quantum dynamics.

I present the theory of observation in two parts. The first part is a presentation of the Born Rule, the subject of this chapter. In the following Chapter 6, I describe the unified dynamic of observation and motion which incorporates the Born Rule (or an equivalent rule, should one arise).

The object of canonical agent observations in quantum automata theory is the Hamiltonian superagent. There are several reasons nature points the theory in this direction. First, each canonical agent has but one set of registers to hold its information. It cannot hold the observed state of another agent without overwriting its own state. Second, agents should respect the superposition principle: observation should be of a superposition of agent-states, not an individual state. Third, an individual canonical coordinate agent has no capacity in itself for "executing the act of observation." That capacity for action belongs to the Hamiltonian superagent as the generator of the time-evolution operator.

*quoted in Robert J. Scully, The Demon and the Quantum (2007), 191.

The Hamiltonian superagent elegantly satisfies the above requirements. It represents the full community of agents as a function of all the agent-states, a superposition. This superposed state is stored in the registers of the Hamiltonian, not the registers of the observing agent. In Chapter 6, I describe the dynamic of motion in which the Hamiltonian's registers are read out by the time-evolution operator to construct the time-evolved state of the observing agent.

The details of the Hamiltonian observation process (setting aside for the moment its intimate relation to the time-evolution operator) are as follows. The Hamiltonian broadcasts its state into the common space-time according to the physics of the force-field. An observing agent, a, exposes multiple receivers—its registers—to the observed Hamiltonian's broadcast. The Hamiltonian's signal falls on the array of exposed registers of the observing agent and excites them. The particular register—and hence the state it will excite—is generally unpredictable and must be described by a probability as the content of the memory register. (But note that in any instance of excitation, only one register will be excited. Statistics emerge from a multitude of excitation trials.) As promised, I adopt the Born rule to describe the physics of unpredictable observation.[†]

The notation for the observation operation of agent h by agent a is

$$(\mathsf{h} \parallel \mathsf{a}),$$

The observation operator takes the observed agent-state, h, as first input and the observing agent-state a as the second. The result of the observation is an agent-state of the same dimensions as the observing and observed agents. The following observation postulate is not restricted to the qubit but applies generally to d-state systems.

Postulate (II. Observation). *(i) In autonomous quantum theory agents do not directly observe one another. Agents observe the Hamiltonian and their fate is determined dynamically by that observed Hamiltonian. (Chapter 6).*

(ii) An agent h *observed by a canonical agent, such as* a*, occurs with a probability* $\mathfrak{p}_{\mathsf{a}j|\mathsf{h}}$*—the conditional probability that the agent in the state* h *will excite the* j*th register of the typical agent such as* a*. This probability is given by the Born Rule, the overlap of agent* h*'s state with* a*'s register, a trace over the product of the two operators,*

$$\mathfrak{p}_{\mathsf{a}j|\mathsf{h}} = \langle \mathsf{a}, j|\mathsf{h}|\mathsf{a}, j\rangle = \mathit{Tr}(|\mathsf{a}, j\rangle\langle \mathsf{a}, j|\,\mathsf{h})\,. \tag{7.1}$$

The register $|\mathsf{a}, j\rangle\langle \mathsf{a}, j|$ *is excited by interaction with* h *to the statistical state*

$$|\mathsf{a}, j\rangle\langle \mathsf{a}, j| \to |\mathsf{a}, j\rangle\mathfrak{p}_{\mathsf{a}j|\mathsf{h}}\langle \mathsf{a}, j|, \qquad j = 1, 2, \ldots, d. \tag{7.2}$$

[†]I leave open the possibility that a more incisive observation postulate may be conceived in future beyond the Born Rule for the construction of the observed agent by an given agent, $(\mathsf{h} \parallel \mathsf{a}(t))$. A different observation rule would modify the Born probability algorithm (7.1) accordingly. But the basic informational structure of Quantum Automata as a theory would remain.

(iii) The full observation consists of a sum of all the excitations of a*'s registers* (7.2) *weighted by their probabilities:*

$$(\mathsf{h} \parallel \mathsf{a}) = \sum_j |\mathsf{a}, j\rangle \mathfrak{p}_{\mathsf{a}j|\mathsf{h}} \langle \mathsf{a}, j|. \tag{7.3}$$

The Born Rule implies this principle:

Principle (Observed State is a Generally Mixed State of the Observer)**.** *The observed state registered by an observing agent is always one or a superposition of the observing agent's eigenstates* (7.3) *with probabilities induced by its interaction with the Hamiltonian superagent*

Canonical agents present different observations among themselves because they are different species. When like species observe one another, the observing agent gets a faithful record of the observed and maximum extraction of information. When unlike species observe one another, they get a random signal and no information is extracted. In the first case, the observed Hamiltonian is written into the registers of the observer. In the second case, the fully random state is written.

One must understand the fully random state as containing all possible states without preference. In interaction with automata in definite states, these states become raw material for new definite states (Chapter 11).

In the illustration below, we imagine the observed agent in each case (the first entry) is a Hamiltonian agent which happens to hold the given state:

$$(\mathsf{q} \parallel \mathsf{p}) = \mathbf{i}, \qquad (\mathsf{q} \parallel \mathsf{q}) = \mathsf{q}, \qquad (\mathsf{p} \parallel \mathsf{q}) = \mathbf{i}, \qquad (\mathsf{p} \parallel \mathsf{p}) = \mathsf{p}.$$

Self-Creation not Self-Observation

‡

‡Richard Feynman, in introducing the principle of least action into Quantum Theory, pointed out the role of measurements and "reduction of the wave packet" —the phenomenon of decoherence: We have presented, in the foregoing pages, a generalization of quantum mechanics applicable to a system whose classical It is possible that an analysis of the theory of measurements is required here. A concept such as the ?reduction of the wave packet? is not directly applicable, for in the mathematics we must describe the system for all times, August 31, 2005 15:31 WSPC/Book Trim Size for 9in x 6in feynman The Principle of Least Action in Quantum Mechanics 69 and if a measurement is going to be made in the interval of interest, this fact must be put somehow into the equations from the start. Summarizing: a physical interpretation should be sought which does not refer to the behaviour of the system at times very far distant from a present time of interest. A point of vagueness is the normalization factor, A. No rule has been given to determine it for a given action expression. This question is related to the difficult mathematical question as to the conditions under which the limiting process of subdividing the time scale, required by equations such as (68), actually converges

The Peres and Wheeler Principles both point to observation as fundamental to the reality of a quantum state. Observers from different species observe different realities. There is, nonetheless, a distinguished state which can be regarded as the agent's 'true' state. What is it?

First, it is not the agent's *self-observed state* according to the foregoing observation dogma for which the agent is both observer and observed:

$$(\mathsf{a} \parallel \mathsf{a}) = \sum_j |a, j\rangle \mathfrak{p}_{\mathsf{a}j|\mathsf{a}} \langle a, j| = \mathsf{a}. \tag{7.4}$$

The observation dogma produces a self-observation (7.4) that has the fixed-point property [25],

$$(((\mathsf{a} \parallel \mathsf{a}) \parallel \mathsf{a}) \cdots \mathsf{a}) = \mathsf{a}.$$

Self-observation touches areas of mathematical physics, logic, and information theory that involve the phenomenon of *self-reference* and its many paradoxes. See the vast literature on the work of Kurt Gödel and AlanTuring. More immediately, see Calude [38, 40], Svozil [44], Szangolies [39], Yanofsky [29]] and the many references cited therein.

Quantum Automata Theory is free from the conundrums of self-reference because observation is integral to dynamics. No self-observations of agents take place save for the self-observation of the Hamiltonian superagent—and that is an invariant. Agents observe, not themselves directly, but the Hamiltonian superagent which dynamically evolves the agent's state of being at each instant of time. This important topic, "Observation as Motion," will be taken up in Chapter 6.)

Origin of the Hamiltonian

The Hamiltonian superagent is an invariant that directs the motion of the canonical agents. What establishes the Hamiltonian? The Hamiltonian, like the initial state of the universe, is an "act of God" outside the bounds of theory. Given the physical interaction of the force field, a system of quanta in given generalized position and momentum initial states comes into existence with a total energy and a corresponding Hamiltonian.

Principle (Hamiltonian Origin and Time Invariance)**.** *(i) The Hamiltonian superagent of a quantum system comes into existence with the system and its associated force laws. (ii) Any operator of a closed quantum system that commutes with the Hamiltonian is a temporal invariant. The Hamiltonian is a temporal invariant as it commutes with itself,*

$$[(\mathsf{h} \parallel \mathsf{h}), \mathsf{h}] = 0.$$

Observation Does Not Distribute over Products of Agent-States

While the qubit automaton is described by a pair of canonical agents q, p whose unitary algebra fixes the third agent as their product

$$\mathsf{s} = i\mathsf{pq}$$

it is not true that an observation of the action by an agent a is a product of observations of position and momentum by that agent

$$(\mathsf{s} \parallel \mathsf{a}) \neq i(\mathsf{p} \parallel \mathsf{a})(\mathsf{q} \parallel \mathsf{a}). \tag{7.5}$$

For example, for agents in the Pauli state (4.11), Rather than the observer gaining information about a complementary agent's state, its observation produces no information whatsoever. Observation does not distribute over products of observables. Self-observations, however, do satisfy the unitary algebra

$$(\mathsf{s} \parallel \mathsf{s}) = i(\mathsf{p} \parallel \mathsf{p})(\mathsf{q} \parallel \mathsf{q}) = \mathsf{s}.$$

Solitary Qubit Observables

The universe of a single qubit is an idealization of a single degree of freedom quantum (such as an electron spin) with vanishing interactions with the rest of the universe. This is a highly idealized object—and an enigmatic one. Each agent bears within itself a reference frame of self-observation. But according to the Wheeler Principle, these frames are solipsistic. An agent acquires a physical existence beyond itself in its observation by other agents. The agents observed may be idealized in this illustration by giving each agent to be observed the role of the Hamiltonian in "free motion." The agents of the physically existent qubit—the observed qubit—is described by the array of nine observables where the diagonal elements represent self-observations and **i** is the fully random agent-state:

$$\begin{matrix} (\mathsf{q} \parallel \mathsf{q}) & (\mathsf{q} \parallel \mathsf{p}) & (\mathsf{q} \parallel \mathsf{s}) \\ (\mathsf{p} \parallel \mathsf{q}) & (\mathsf{p} \parallel \mathsf{p}) & (\mathsf{p} \parallel \mathsf{s}) \\ (\mathsf{s} \parallel \mathsf{q}) & (\mathsf{s} \parallel \mathsf{p}) & (\mathsf{s} \parallel \mathsf{s}) \end{matrix} \quad = \quad \begin{matrix} \mathsf{q} & \mathbf{i} & \mathbf{i} \\ \mathbf{i} & \mathsf{p} & \mathbf{i} \\ \mathbf{i} & \mathbf{i} & \mathsf{s} \end{matrix} \tag{7.6}$$

The corresponding observed agent-actors are

$$\begin{matrix} (\mathsf{Q} \parallel \mathsf{q}) & (\mathsf{Q} \parallel \mathsf{p}) & (\mathsf{Q} \parallel \mathsf{s}) \\ (\mathsf{P} \parallel \mathsf{q}) & (\mathsf{P} \parallel \mathsf{p}) & (\mathsf{P} \parallel \mathsf{s}) \\ (\mathsf{S} \parallel \mathsf{q}) & (\mathsf{S} \parallel \mathsf{p}) & (\mathsf{S} \parallel \mathsf{s}) \end{matrix} \quad = \quad \begin{matrix} \mathsf{Q} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathsf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathsf{S} \end{matrix} \tag{7.7}$$

where **0** is the null operator. The array of nine entries summarizes the correlations between observed and observer. The definite correlations populate the self-diagonal portraying the classical state of the qubit while the off-diagonal correlations exhibit its completely random quantum correlations.

Chapter 8

Observation as Motion: Paradigm of Autonomous Quantum Theory

I now bring together the Born theory of observation (or a possible future equivalent) with a proposal for the way in which nature puts that observation to use: observation, understood as force-field signal reception, triggers motion. Observation and motion are one. This perspective, that assigns an essential role to observation, is consistent with theories of time that rest on an essential role for observation, such as that of Page and Wootters [22]. I begin with a review of the unitary theory of quantum motion.

Motion in Unitary Quantum Mechanics

In the unitary theory of motion, there exists a quantity of motion, the action $\mathcal{S}$, which is a functional of agent-states generated by the time-path integral of the Hamiltonian superagent:

$$\mathcal{S} = \int \mathsf{H}(\mathsf{q}, \mathsf{p})dt.$$

The Hamiltonian considered here is that for a closed system and is time -invariant. Action instantiated by the Hamiltonian both constructs and directs the time-evolution operator

$$\mathsf{U}(t) = e^{(i/\hbar)\mathcal{S}} = e^{(i/\hbar)\mathsf{H}t}$$

as its generator. A time-dependent operator, such as $\mathsf{q}(t)$, evolves unitarily over a time-interval t, t' as

$$\mathsf{q}(t') = e^{-(i/\hbar)\mathsf{H}(t'-t)}\mathsf{q}(t)e^{i/\hbar)\mathsf{H}(t'-t)}. \tag{8.1}$$

Other agent-states, such as $\mathsf{p}(t)$ and $\mathsf{s}(t)$, have dynamically evolving matrices according to the a same algorithm.*

The motion is plainly simple. It consists of rotations in Hilbert space with frequencies proportional to the eigenvalues of the Hamiltonian.

As is well known, this theory is incomplete. It must be supplemented with various proposals for decohering quantum states beginning with von Neumann's procedure which entwines periods of unitary evolution with periods of state function collapse. None of these decohering procedures including the ongoing development of quantum operations [4] in open quantum systems are part of the present theory. Automata theory, however, can accommodate open quantum systems theory by regarding each agent as an open quantum system in the environment of other agents as described below.

Motion as Computation

Principle (Automata Compute). *Interacting qubits are automata with computational capability, natural computers which operate on input and produce output. The force-field provides the hardware for the computation. Agent-states provide the memory registers. Naturally embedded code in the informational counterpart of energy—the Hamiltonian—and a master code covering all matter—equations of motion—provide the operating system. Force fields instantiate the operating system in a mechanical processor—the time-evolution operator, an analog of central or distributed processors in conventional computer architectures.*

Quantum automata compute themselves. The automaton's own state at any time is part of the input. The output of the computation is each automaton's new state in time. Because of the random elements in an agent's observation of the Hamiltonian, this natural computational process is partly indeterministic, partly deterministic.

The process of motion may be described in the idiom of modern computers without implying that these are precisely the operations nature employs. (It's clear, for example, that the von Neumann architecture, overwhelmingly employed in modern computer systems, is not the architecture of the brain, nervous system, and its associated chemical transport and computational circuits). I do imply that nature's process is similar in its essentials of reading present states in a natural network, such as a biochemical network (with some random bits as required by quantum uncertainty) as 'input,' and evolving them to new states as 'output.' The transformation of input states to output states is the essence of computation.

*This is another instance of quantum agents "evolving as their own basis."

Automata Time-Evolution

Principle (Information in Time-Evolution)**.** *The way a given agent obtains information on the states of other agents is by observation/interaction with them via the force field in time-evolution. The states of other agents make themselves known (act on) an observing agent in its observation of the Hamiltonian superagent.*

Accordingly, the dynamic process itself in which agents advance in time is an observational process in which agents observe the Hamiltonian and, in the process, are themselves evolved to new states dictated by the observation. The result of their observation appears as their new state in time. Here are the details.

> The Hamiltonian state is stored in its registers $|\mathsf{h}, j\rangle\langle\mathsf{h}, j|$ as its time-invariant agent-state h which corresponds to its agent-actor presentation $\mathsf{H} = 2\mathsf{h} - \mathsf{I}$. An agent $\mathsf{a}(t)$ observes the Hamiltonian agent as agent-state h and agent-actor
>
> $$(\mathsf{H} \parallel \mathsf{a}(t)) = 2(\mathsf{h} \parallel \mathsf{a}(t)) - \mathsf{I}.$$
>
> This observed Hamiltonian simultaneously acts as the generator of the time-evolution operator which dynamically writes the observing agent's time-advanced state into the registers of $\mathsf{a}(t)$ updating it to $\mathsf{a}(t')$.
>
> $$\mathsf{a}(t') = e^{-(i/\hbar)(\mathsf{H} \parallel \mathsf{a}(t))(t'-t)}\mathsf{a}(t)e^{(i/\hbar)(\mathsf{H} \parallel \mathsf{a}(t))(t'-t)}$$
>
> In other words, the information contained in the Hamiltonian state h and its actor presentation H observed by typical agent $\mathsf{a}(t)$
>
> $$(\mathsf{H} \parallel \mathsf{a}(t)) = 2(\mathsf{h} \parallel \mathsf{a}(t)) - \mathsf{I}.$$
>
> appears in $\mathsf{a}(t')$ in the construction of the time-advanced state. Observation and evolution are proposed to occur in the same process.

The Hamiltonian superagent is fixed by the positions and momenta of the particles according to their instantaneous configurations and the force law. But it is physically manifest in observations of the community of agents. Automata theory comes equipped with an agent communication and observation dogma described previously.

The centrality of observation is captured in the third postulate of Autonomous Quantum Theory:

Postulate (III. Evolution)**.** *The Hamiltonian directing the time-evolution of a given agent* a *is the Hamiltonian observed by that agent,* $(\mathsf{H} \parallel \mathsf{a})$.[†]

Under this postulate an agent's observed Hamiltonian will not generally be the same as the canonical time-invariant Hamiltonian. The agent-observed Hamiltonian will generally be open to random data and the motion it directs will correspondingly be partly random. The motion cannot be fully random because a qubit automaton does possess 1 bit of definite information (the definite momentum agent in free motion, for example).

Motion consists of an agent's observation of the Hamiltonian with the observation appearing (stored) as the time-evolved state in the agent's registers. This is conventional quantum dynamics except for a major difference: the Hamiltonian directing the agent's motion is not the time-invariant Hamiltonian (which directs the Hamiltonian agent's own motion), but rather the agent-observed Hamiltonian. Driven by Hamiltonian code, the agent's observation changes its state according to the time-evolution operator executing the Hamiltonian code observed by that agent.

Information from other agents (their positions and momenta) makes itself known to the observing agent by its impact on the observing agent's time-evolved state, $\mathsf{a}(t+\delta t)$. The *effect, the influence of other agents* is stored in the agent's registers as its time-evolved state, not as a copy of other agents' states in their energy contributions to the Hamiltonian.

The time-evolution operator overwrites the agent's register with the new time-evolved state. Reading the existing state transfers the information to the time-evolution operator. This is the nature of a dynamic in which observation and evolution are seamlessly part of the same process. In the following chapter I present a key theorem that describes the stationary statistics of quantum information flow from observed Hamiltonian to the time-advanced state of an observing agent.

Equations of Motion

Like all agents, the Hamiltonian superagent exhibits its reality in its observations by other agents according to the Wheeler Principle. The formal time-evolution directed by the Hamiltonian of a quantum observable $\mathsf{a}(t)$ is

[†]Note that this postulate is not a statistical operation applied to the time-evolution operator, viz

$$\mathsf{a}(t') \neq (e^{-(i/\hbar)\mathsf{H}(t'-t)\mathsf{a}(t)}\mathsf{a}(t)e^{(i/\hbar)\mathsf{H}(t'-t)} \parallel \mathsf{a}(t)),$$

$$\frac{d}{dt}\mathsf{a} = (i/\hbar)[\mathsf{a}, \mathsf{H}].$$

According to the Evolution Postulate III, the equation of motion of a quantum agent is governed by the agent-observed Hamiltonian $\mathsf{H} \to (\mathsf{H} \parallel \mathsf{a})$ whose commutator with the agent vanishes

$$\frac{d}{dt}\mathsf{a} = (i/\hbar[\mathsf{a}, (\mathsf{H} \parallel \mathsf{a})] = 0.$$

Agent-states are therefore statistically constant in time. But they are not fully random because the qubit automaton contains 1 bit of definite information. This 1 definite bit may be the provenance of a single agent or it may be shared among the three

Exemplary Motion: Free Motion

Examine a time-evolution for a free motion Hamiltonian for which

$$\mathsf{H} = \mathsf{P}, \qquad \mathsf{h} = \mathsf{p}$$

and see what happens If the operator agents are assumed omniscient—each agent (including the canonical agents and the Hamiltonian) has access to all other agent's information. In particular, on the basis of this assumption (which is false in Automata Theory) the position and action agents receive a definite signal in observing the Hamiltonian $\mathsf{H} = \mathsf{P}$ just as the momentum agent received a definite signal. That definite signal leaves the momentum agent fixed in time since it commutes with the Hamiltonian.

$$\frac{d\mathsf{p}}{dt} = (i/\hbar)[\mathsf{p}, (\mathsf{H})_{\mathsf{p}}] = (i/\hbar)[\mathsf{p}, \mathsf{P}] = 0, \qquad \mathsf{p}(t) = \mathsf{p}(0).$$

The dynamics of the other two agents are‡

$$\frac{d\mathsf{q}}{dt} = -\frac{1}{\hbar}(2\mathsf{s} - \mathsf{I}).$$

$$\frac{d\mathsf{s}}{dt} = -\frac{1}{\hbar}(2\mathsf{q} - \mathsf{I}).$$

While the Hamiltonian agent and momentum are constant in time, the position and action execute coupled rotations in Hilbert space. (Note that since the action agent

‡The details of coupled rotations of omniscient agents are

$$\frac{d\mathsf{q}}{dt} = (i/\hbar)[\mathsf{q}, \mathsf{H}] = (i/\hbar)[\frac{1}{2}[\mathsf{I} + \mathsf{Q}], \mathsf{H}] = (i/\hbar)[\frac{1}{2}[\mathsf{Q}, \mathsf{P}] = -\frac{1}{\hbar}\mathsf{S} = -\frac{1}{\hbar}(2\mathsf{s} - \mathsf{I}).$$

$$\frac{d\mathsf{s}}{dt} = (i/\hbar)[\mathsf{s}, \mathsf{H}] = (i/\hbar)[\frac{1}{2}[\mathsf{I} + \mathsf{S}], \mathsf{P}] = (i/\hbar)[\frac{1}{2}[\mathsf{SP}] = -\frac{1}{\hbar}\mathsf{Q} = -\frac{1}{\hbar}(2\mathsf{q} - \mathsf{I})$$

is fixed by position and momentum, it could have been eliminated giving a single second-order equation for the position q.)

In free motion, the qubit position agent continuously rotates with a frequency $\omega \equiv |\mathsf{H}|/\hbar$. The time evolution operator $e^{-i\hat{\mathsf{P}}\omega t}$can then be expressed non-dimensionally in the Pauli state (4.12).

$$\hat{\mathsf{P}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Let its initial state be the position Pauli state

$$\mathsf{q}(0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

The motion is then

$$\mathsf{q}(t) = e^{-(i\hat{\mathsf{P}}\omega t}\mathsf{q}(0)e^{(i\hat{\mathsf{P}}\omega t}$$

At time $t = \pi/\omega$, the position agent will be in the bit-flipped state

$$\mathsf{q}(\pi/\omega) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Now turn to a description of the same motion in automata theory. We have already seen that it is axiomatic that this system evolves with 1 definite bit and 1 random bit. No definite theory, as was the case for omniscient agents, is possible for them. The theory must be statistical. The Hamiltonian $\mathsf{H} = \mathsf{P}$ has observations by each agent as was shown previously

$$(\mathsf{H} \parallel \mathsf{q}) = (\mathsf{P} \parallel \mathsf{q}) = \mathbf{0}, \qquad (\mathsf{H} \parallel \mathsf{p}) = (\mathsf{P} \parallel \mathsf{p}) = \mathsf{P}, \qquad (\mathsf{H} \parallel \mathsf{s}) = (\mathsf{S} \parallel \mathsf{s}) = \mathbf{0}.$$

We see immediately that the momentum agent—as the Hamiltonian—is an operator invariant in time. The particle's momentum machine $\mathsf{p} = const$ runs at a fixed setting (without flipping its state),

$$\frac{d\mathsf{p}}{dt} = (i/\hbar)[\mathsf{p}, (\mathsf{H})_{\mathsf{p}}] = (i/\hbar)[\mathsf{p}, \mathsf{P}] = 0, \qquad \mathsf{p}(t) = const.$$

The momentum agent holds the qubit automaton's 1 definite bit of information: the value $\mathsf{p} = const$. Although $\mathsf{p}(t)$ is a 2×2 matrix with two projectors, one bit of information distinguishes which of those projectors is the invariant and its self-observation affirms it. The Hamiltonian (as the momentum) exhausts 1 bit of definite information of the qubit automaton. The remaining position and action agents therefore evolve completely randomly.

According to Observation Postulate II, observation of this free-motion Hamiltonian by agents complementary to the momentum agent are fully random states while the observation by the momentum agent is a self-observation :

$$(\mathsf{h} \parallel \mathsf{q}) = \mathbf{i}$$

$$(\mathsf{h} \parallel \mathsf{p}) = \mathsf{p}$$

$$(\mathsf{h} \parallel \mathsf{s}) = \mathbf{i}$$

where $\mathbf{i} = \frac{1}{2}\mathsf{I}$ is the fully random state. The corresponding agent-actors are

$$(\mathsf{H} \parallel \mathsf{q}) = \mathbf{0}$$

$$(\mathsf{H} \parallel \mathsf{p}) = \mathsf{P} = 2\mathsf{p} - \mathsf{I}$$

$$(\mathsf{H} \parallel \mathsf{s}) = \mathbf{0}$$

The position and action agents are described by their expectation values $(\mathsf{q} \parallel \mathsf{q}), (\mathsf{s} \parallel \mathsf{s})$. Their Hamiltonian observations, $(\mathsf{H} \parallel \mathsf{q})$, $(\mathsf{H} \parallel \mathsf{s})$, produce the null operator and the statistics evolve at 'null rates':

$$\frac{d}{dt}(\mathsf{q} \parallel \mathsf{q}) = (i/\hbar)[(\mathsf{q} \parallel \mathsf{q}), (\mathsf{H} \parallel \mathsf{q})] = \mathbf{0},$$

$$\frac{d}{dt}(\mathsf{s} \parallel \mathsf{s}) = (i/\hbar)[(\mathsf{s} \parallel \mathsf{s}), (\mathsf{H} \parallel \mathsf{s})] = \mathbf{0},$$

In contrast to the momentum $\mathsf{p} = const$ which is in a definite and invariant state, the position and action agents execute completely random motion described by the fully random statistical state $\mathbf{i}$:

$$\mathsf{q}(t) = \mathsf{q}(0) = \mathbf{i} = \frac{1}{2}\mathsf{I}, \qquad \mathsf{s}(t) = \mathsf{s}(0) = \mathbf{i} = \frac{1}{2}\mathsf{I}.$$

These statistical states are stationary but, of course, they do not generally correspond to any instantaneous state which will be a random 2×2 unit trace density (and thereby Hermitian) matrix. Unlike the evolutions of omniscient agents which rotate at definite frequencies in Hilbert space, the agents in automata theory rotate with one definite bit and one random bit.

In the case of the free-motion Hamiltonian, that random bit is distributed between the position and action agents. If the definite bit is assigned to the position agent, that agent rotates in a fixed direction (say clockwise) but the momentum agent rotates randomly (switching randomly between clockwise and counter-clockwise motion) and yielding a statistical average of "stasis."

Exemplary Motion: Little Heisenberg Hamiltonian

A second exemplary Hamiltonian is the single qubit *'little Heisenberg'* Hamiltonian. This Hamiltonian is an equal superposition of all three canonical agents,

$$\mathsf{H} = (\mathsf{Q} + \mathsf{P} + \mathsf{S})/3, \qquad \mathsf{h} = (\mathsf{q} + \mathsf{p} + \mathsf{s})/3.$$

The agent-observed Hamiltonians consist of both self-observations and complementary observations.[§] The definite information of 1 bit in $\mathsf{q}, \mathsf{p}, \mathsf{s}$ is distributed equally over the three agents making up the Hamiltonian along with 1 bit of random information:

$$(\mathsf{h} \parallel \mathsf{q}) = \frac{1}{3}\mathsf{q} + \frac{2}{3}\mathbf{i}, \quad (\mathsf{h} \parallel \mathsf{p}) = \frac{1}{3}\mathsf{p} + \frac{2}{3}\mathbf{i}, \quad (\mathsf{h} \parallel \mathsf{s}) = \frac{1}{3}\mathsf{s} + \frac{2}{3}\mathbf{i}. \tag{8.2}$$

The self-observed Hamiltonian in this case is a mixed state—but not a fully random state $\mathbf{i}$:

$$(\mathsf{h} \parallel \mathsf{h}) = \mathsf{h} = (\mathsf{q} + \mathsf{p} + \mathsf{s})/3.$$

The corresponding observed Hamiltonian agent-actors adopt the agent-actor of the observing agent

$$(\mathsf{H} \parallel \mathsf{q}) = \frac{1}{3}\mathsf{Q}, \quad (\mathsf{H} \parallel \mathsf{p}) = \frac{1}{3}\mathsf{P}, \quad (\mathsf{H} \parallel \mathsf{s}) = \frac{1}{3}\mathsf{S}.$$

Hamiltonian self-observations for the little Heisenberg Hamiltonian can be illustrated for agents in the Pauli state (6.4). They are a mixture of the pure state automata $\hat{\mathsf{h}}_1, \hat{\mathsf{h}}_2$ of Eqs (6.5) with probabilities,

$$\mathfrak{p}_{\mathsf{h}1} = \frac{1}{6}(3 + \sqrt{3}), \qquad \mathfrak{p}_{\mathsf{h}2} = \frac{1}{6}(3 - \sqrt{3}).$$

This statistical state (6.4) is mixture of the states (6.5) with purity $\mathsf{Tr}\left(\mathsf{h}^2\right) = 2/3$.

Observations of agents in in Pauli states under the little Heisenberg Hamiltonian are likewise mixed states. The position and and action agent observations of the Hamiltonian (6.4) are

$$(\mathsf{h} \parallel \mathsf{q}) = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}, \qquad (\mathsf{h} \parallel \mathsf{p}) = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{2} \end{pmatrix} \qquad (\mathsf{h} \parallel \mathsf{s}) = \begin{pmatrix} \frac{1}{2} & \frac{1}{6}i \\ -\frac{1}{6}i & \frac{1}{2} \end{pmatrix},$$

In any instance of interaction the instantaneous observed Hamiltonian will be one or the other of $\mathsf{h}_1, \mathsf{h}_2$, but it is not definite (not repeatable under fixed conditions). The hamiltonian self-observed is, of course, a temporal invariant.

Evolution under the little Heisenberg Hamiltonian is stasis. The equation of motion for any of the agents, exemplified by q, is the statistical stationary state

$$\frac{d\mathsf{q}}{dt} = (i/\hbar)[\mathsf{q}, (\mathsf{H})_{\mathsf{q}}] = \mathbf{0}, \qquad \mathsf{q}(t) = const,$$

and similarly for the other agents. These agents hold their Pauli states (4.11) in self-observation, but their observation in the Hamiltonian by other complementary agents contain random data, an example of the Peres Observation Principle.

[§]The details are:

$$(\mathsf{h})_{\mathsf{q}} = \frac{1}{3}[(\mathsf{q} \parallel \mathsf{q}) + (\mathsf{p} \parallel \mathsf{q}) + (\mathsf{s} \parallel \mathsf{q})] = \frac{1}{3}\mathsf{q} + \frac{2}{3}\mathbf{i}$$

Emergent New Information

If agents were omniscient and had complete access to one another's information, the observation of agent a's state of being by agent b would be identical to a's dynamic state. This is not the case in automata theory because agent b only has access to agent a's state within the community of states registered in the Hamiltonian. This accounts for the fundamental differences between information acquisition and transmission in quantum systems compared to classical.

The act of observation introduces new information into a quantum system because some of the observed signal from the Hamiltonian by each agent may be random. Random information is new information; it was not produced by arithmetically processing previous state information (von Neumann's sin). Since the agent acts on that information dynamically, the randomly acquired information has dynamic consequences. It becomes entrained in the evolutionary flow of quantum motion. Random data appearing in agent observations is the source of new information flow into nature.

Principle (Emergent Information)**.** *New information enters a quantum system brought in by agent evolution directed by an observed Hamiltonian with some random data.*

Abundant Randomness

Readers must be struck by the copious amounts of randomness produced through quantum observation in the foregoing illustrations. The reason for this is this illustration is for an isolated single degree of freedom. An isolated qubit automaton is, in a rough way of speaking, "half-random, half-definite" and the universe of an isolated qubit automaton a "half-random, half-definite" universe.

My narrative thus far without further elaboration is most naturally applicable to a hot, dense gas in which otherwise structureless particles travel over free paths between collisions as force fields act: a gas of qubit automata. Such a gas is indeed beset with a high degree of randomness. Unstable collision manifolds magnify the quantum randomness of agent interaction/observations.

To go beyond the universe of the qubit one must take into account the way degrees of freedom reestablish themselves in different phases of matter and the nature of the force fields that sustain them. The formation of condensed matter and the containment of the far-ranging translational degrees of freedom in a gas within the far more restricted amplitudes of roton and vibratory modes in condensed matter is a critical process in the creation of order in the universe (Chapter 8). Chemical bonding further sets natural "containing structures" in place to bring further order into matter.

Time Scales

The time scales for observation and execution of motion are set by the physics of the Hamiltonian, in particular, the energy levels of the motion. The Hamiltonian of a complex system may contain many energy levels and time scales. Let us focus on a single energy scale $|\mathsf{H}|$. The characteristic time for the process represented by that Hamiltonian is

$$\tau \approx \hbar/|\mathsf{H}|.$$

Some typical examples are,

$$\text{For molecular bond vibrations} \qquad \hbar/|\mathsf{H}| \approx 10^{-15} sec.$$

$$\text{For flagella rotations} \qquad \hbar/|\mathsf{H}| \approx 10^{-1} - 10^{-2} sec.$$

$$\text{Motor protein walking} \qquad \hbar/|\mathsf{H}| \approx 1 sec.$$

Generalized Observations

We have seen that observation is intrinsic to the dynamic of motion. The result of an observation of the Hamiltonian by an agent is to be found int the next state in time to which the agent advances. Suppose we rather instead ask for the observation of one agent-state by another. With a single qubit, the observed agent must be the Hamiltonian which has already been demonstrated. For more general observations one must go beyond a single qubit to tensor automata. For multiple qubits, the agents of one qubit may observe the agents of another. This will include the richness of entanglement. (See Chapter 10, *Tensor Automata*).

A Coding Theorem

The encoding of information in computers is a familiar operation. A hardware register of some construction based on a physical effect (transistor semiconductor levels, atomic spins, etc.) holds the information. Let us say the bits in a particular hardware register are set to encode the letter "*A*" in some code, say an 8 bit register in ASCII code. When the bits in the register are set to 01000001, they encode the letter "*A*." But these bits also represent a state of nature, a set of atoms in a containing structure with its electronic states or magnetic spins in a certain state. This is the "natural" state of the physical matter independent of our regarding them as code for a letter of the alphabet. Yet this same set of spins also represents the letter state A according to the encoding design of the register. How can the same register encode two quite different states, say transistor levels or physical spins on the one hand, and the letter "A" on the other? How does nature manage this dual representation?

The answer is in the dynamics that set the register according to a Hamiltonian designed to encode the "A". The particular design Hamiltonian, for example, acts on a register in a standard state (say all bits set to "0") and dynamically advances them to the bit pattern 01000001 for "A". The dynamic of motion, the action of the Hamiltonian in advancing the states from the standard state to the encoded "A" has set up a correspondence between the physical states of the transistor or spins and the code of the letter "A". Dynamic processes of nature yoked to the design of a computer register bring forth the coincidence of a particular set of physical spins and the letter "A".

This process is fundamental to nature's capacity for agents to form *representations* of other agent-states in their own physical states. The Hamiltonian that executes this process is the critical part of the encoding.

Summary

The theory of quantum automata rests on individual canonical agents ("positions and momenta"), a canonical pair for each degree of freedom of the underlying force field. Agents observe and gain information about other agents, not individually, but in concert in their superposed presence in the Hamiltonian—a global superagent. Agent observed Hamiltonians are postulated to follow well-established quantum protocols of the Born Rule. This means agents hold Hamiltonian observations that may differ from one another and these differences may be random.

The time-evolution operator acting on each agent is postulated to be the time-evolution operator directed by the Hamiltonian observed by that agent. In this manner, a stream of random information enters quantum dynamics. An important instance of this postulate is the Hamiltonian operator-agent itself whose evolution is directed by its self-observation. The Hamiltonian self-observation is faithful and evolves unitarily without random input though this is not necessarily the case for other agents.

Quantum agency embodies both subjective and objective traits. An agent's state is stored in its own register—a clear instance of subjectivity. The states of objects (states of other agents) are stored as the state of the community of agents in the Hamiltonian's registers—a superposition of all the agent-states, a quantum 'objectivity.'

Subjects learn about objects as the observed state of the community of agents. Objectivity is a communal concept. Objectivity arises from the presence of multiple individual agent perspectives in the Hamiltonian superposition. All agents have a voice in the object that codes their evolution—the Hamiltonian, the superagent representing the community of other agents. "Subjects learn about other agents" in the scenario above by observing the Hamiltonian. Their observation is integral to and occurs in the mechanics of motion. The result of their observation is revealed in the next state to which the time evolution operator (executing their observed Hamiltonian) moves them.

Quantum agents are ceaselessly acting, observing and moving according to the results of observation in a universe in which time is unstoppable.

Chapter 9

The Fundamental Theorem: Statistical Stasis

The global state of a quantum system is a superposition of individual agent-states held in the registers of the Hamiltonian. Agents obtain Information on these states from observation of the Hamiltonian. According to the Born rule, the agent-observed Hamiltonian is stored in the projectors of the observing agent. The agent-observed Hamiltonian and the observing agent therefore commute. The result is a null argument for the time-evolution operator.

Theorem (Statistical Stasis)**.** *An agent-observed Hamiltonian is in the same basis* (7.3) *(possesses the same projectors) as the observing agent according to the Born Rule. The agent therefore commutes with its observed Hamiltonian,*

$$[(\mathsf{H} \parallel \mathsf{a}), \mathsf{a}] = \mathbf{0}.$$

The vanishing commutator with the Hamiltonian in the time-evolution operator produces a stationary statistical state,

$$\mathsf{q}(t + \delta t) = (i/\hbar)[\mathsf{q}(t), (\mathsf{H} \parallel \mathsf{q}(t))]\delta t + \mathsf{q}(t) = \mathsf{q}(t).$$

Theory confirms a statistical stationary state, but it provides no information about the detailed dynamics of the agent. Although the statistical state is stationary, the time-evolved state is one of dynamic fluctuations inaccessible to theory.

Statistical Stasis vs. Time-Invariance

A Hamiltonian of a sufficiently different species from a given agent will be observed by that agent as a fully random agent-state $\mathsf{h} = \mathbf{i}$ with a null agent-actor $\mathsf{H} = \mathbf{0}$. A null observed Hamiltonian directs a stationary random process in the time-evolution operator with the instantaneous states making up the statistical state fluctuating in time.

Principle (Statistical Stasis vs. Absolute Time-Invariance)**.** *An agent that observes the Hamiltonian as a fully random state* h *with a null-operator actor* H *is predicted to behave in statistical stasis (time-invariant expectation values) as indicated by the time-evolution operator with* $\mathsf{H} = \mathbf{0}$ *which, perforce, gives a null-operator commutator with any operator upon which it acts:*

$$\mathsf{H} = \mathbf{0} \Rightarrow \mathsf{Ha} = \mathsf{aH} = \mathbf{0}.$$

$$\Rightarrow [\mathsf{H}, \mathsf{a}] = \mathbf{0}.$$

This is in contrast to an absolute stationary state for operators which commute "instantaneously" with a non-vanishing Hamiltonian,

$$\mathsf{H} \neq \mathbf{0}. \quad [\mathsf{H}, \mathsf{a}] = \mathbf{0}.$$

$$\frac{d}{dt}\mathsf{a} = \frac{i}{\hbar}[\mathsf{H}, \mathsf{a}] = \mathbf{0}, \qquad \mathsf{a}(t) = const.$$

The stasis of an agent described by a fully random agent-state (4.19) *is not instantaneously constant but fluctuates randomly. This is the difference between* $(\mathsf{H} \parallel \mathsf{a}) = \mathbf{0}$ *whose commutator with agent* a *identically vanishes because the Hamiltonian vanishes* $[\mathbf{0}, \mathsf{a}] \equiv \mathbf{0}$ *and a non-vanishing Hamiltonian that commutes with* a *because it is a temporal invariant,* $[\mathsf{H}, \mathsf{a}] = \mathbf{0}$*. In one case we have a statistical invariant with fluctuating agent-states, in the other a temporal invariant with absolutely stationary states.*

Heisenberg Uncertainty Principle

The theory of quantum automata naturally reproduces Heisenberg's original discovery of the uncertainty principle in the dispersion relationship of position and momentum. The dispersion of the observations of agent a by agent b is a matrix whose values are dispersed from those of a by the observing action of b:

$$\Delta(\mathsf{a}, \mathsf{b}) \equiv \sqrt{(\mathsf{a}^2 \parallel \mathsf{b}) - (\mathsf{a} \parallel \mathsf{b})^2}.$$

(Note that dispersions are properties of observed agent-states but are not necessarily density matrices.)

Self-observations are always non-dispersive and observations by complements are maximally dispersive. Dispersions are illustrated by the canonical pair for which dispersions are the same whether q observes p or p observes q:

$$\Delta(\mathsf{q}, \mathsf{q}) = 0 \qquad \Delta(\mathsf{q}, \mathsf{p}) = \frac{1}{2}\mathsf{I}, \qquad \Delta(\mathsf{p}, \mathsf{q}) = \frac{1}{2}\mathsf{I}, \qquad \Delta(\mathsf{p}, \mathsf{p}) = 0.$$

Complement dispersions cannot be reduced below a minimum measured by the matrix norm: the Heisenberg Principle,

$$\| \Delta(\mathsf{q},\mathsf{p}) \| \geqslant \frac{1}{2}, \qquad \| \Delta(\mathsf{q},\mathsf{p})\Delta(\mathsf{p},\mathsf{q}) \| \geqslant \frac{1}{4}, \qquad \| \Delta(\mathsf{p},\mathsf{q}) \| \geqslant \frac{1}{2} \tag{9.1}$$

Let q_o, p_o be the physical observables of a quantum particle, for example, the ground state radius and orbital momentum of the hydrogen atom satisfying

$$p_o q_o = \hbar.$$

Dimensioned agents $\bar{\mathsf{q}}, \bar{\mathsf{p}}$ may then be represented at this microscopic scale in terms of the non-dimensional magnitudes q_o, p_o as macro-variables

$$\bar{\mathsf{q}} = q_o \mathsf{q}, \qquad \bar{\mathsf{p}} = p_o \mathsf{p}.$$

The physical dispersion of the system in dimensioned coordinates is the Heisenberg relation (9.1)

$$\| \Delta(\bar{\mathsf{q}},\bar{\mathsf{p}}) \| \geqslant \frac{1}{2}\hbar, \qquad \| \Delta(\bar{\mathsf{q}},\bar{\mathsf{p}})\Delta(\bar{\mathsf{q}},\bar{\mathsf{p}}) \| \geqslant \frac{1}{4}\hbar, \qquad \| \Delta(\bar{\mathsf{p}},\bar{\mathsf{q}}) \| \geqslant \frac{1}{2}\hbar$$

Classicality: Numbers vs. Smallness

Generally speaking, quantum effects, such as superposition and entanglement, are not directly observable at classical scales. This scale is large compared to the microscopic and nanophysical scale of atoms and molecules. The emphasis on "largeness" of the system in mass, length, etc. of the particles as the essence of classical behavior, while true, is misleading. The crucial element of quantum behavior is that *few quanta are present to decohere one another.*

Principle (Large Numbers of Interacting Quanta are Most Classical-like)**.** *Since atoms, molecules, and other nanophysical objects have such small mass and length scales, it takes large numbers of them to reach macroscopic size. But large numbers of interacting quantum agents are precisely what are required to strongly decohere observational dynamics. It is the number of interacting particles that is the driving factor of classicality.*

As illustration, let a quantum system consist of the condensation of $\mathcal{N}$ qubits into a massive single degree of freedom qubit corresponding to a bound massive particle with but one translational freedom and a momentum $\mathcal{N}\bar{\mathsf{p}}$. One can imagine a gas of $\mathcal{N}$ atoms condensing into a single liquid droplet moving in a single degree of freedom.* The dispersion relationship for the condensate is then

$$\Delta(\bar{\mathsf{q}}, \bar{\mathsf{p}})\Delta(\bar{\mathsf{q}}, \bar{\mathsf{p}}) \geqslant \frac{\hbar}{4\mathcal{N}}\mathsf{I}$$

Dispersion declines with the number of condensed degrees of freedom. In typical macroscopic systems this is a number of the order the Avogadro number and the dispersion is correspondingly negligible in most macroscopic instances: the condition of classicality. Technology seeking to exploit quantum effects, such as humanly constructed quantum computers, exists close to the Heisenberg limit

$$\| \Delta(\bar{\mathsf{q}}, \bar{\mathsf{p}}) \| \approx \frac{\hbar}{2\mathcal{N}},$$

In a macroscopic world, trillions upon trillions and more degrees of freedom are available to fashion apparatus that selectively isolates only a few quantum freedoms. The ignorable freedoms may be in glass and metal mechanical parts, wires, switches, lasers and their power supplies, forming an ion trap with an interferometer set-up, or other quantum experiment systems. These macroscopic objects manipulate the few degrees of quantum freedom of the trapped ions or atoms with carefully crafted Hamiltonians. In the macroscopic world of ordinary life, the loss of a few skin or hair cells from the body is unnoticeable. At the quantum nanoscale of two or three qubits, the loss of one of the qubits is a catastrophe.

Quantum evolution may then be dramatically different from that of the isolated qubit in which one random bit accompanies every definite bit. In these highly macroscopic systems, definite observations of the few quantum freedoms that constitute the system is possible while ignoring the vastly greater number of random interactions trapped in the solid states of condensed matter in glass and metal of the apparatus and beyond that in the environment.

Automata Theory and the Feynman Path Integral

The postulates of Autonomous Quanta endow quantum dynamics with a statistical behavior like that produced by the Feynman path integral. In the Feynman Path Integral the statistics of quantum motion arise from the freedom of particles to explore all possible trajectories from a state $(\mathsf{q}, \mathsf{p})|_{t_1}$ to $(\mathsf{q}, \mathsf{p})|_{t_2}$ along a path $\int dt$ over the interval $t_2 - t_1$ according to its action $\mathcal{S} = \int \mathsf{H}dt$, each path weighted by the factor $Ae^{(i/\hbar)\mathcal{S}}$.†

*A 50 microliter water drop contains approximately 10^{21} (sextillion) water molecules

†The action $\mathcal{S}$ appearing in the path integral should not be confused with the dependent agent S of the qubit automaton.

Action has an oscillating phase along a path. The superposition of paths, each with a different oscillating probability amplitude, produces phase interference which destroys and reinforces paths contributing to the sum. The result of a summation (superposition) over all possible paths produces the most likely or expected path which is identified as the 'classical' path.

> Individual agents in automata theory do not receive definite signals in general from the Hamiltonian. The instantaneous evolutions of those indefinite observations drive different quantum trajectories that make up the statistical observed states $(\mathsf{H} \parallel \mathsf{q}), (\mathsf{H} \parallel \mathsf{p})$,. Summation over of all possible paths in the FPI for a given observable —agent in the language of automata theory—corresponds to the summation of all possible observations in the expectation value of the Hamiltonian observed by that agent, such as a $(\mathsf{H} \parallel \mathsf{q})$. These differently observed Hamiltonians and the uncertain quantum trajectories they direct correspond to the different paths of the Feynman Path Integral.

Chapter 10

Emergence of the Classical World

Multiple observers dense matter common decoherence.

Condensed matter key to a classical universe

effective complexity: fully random state is not effectively complex. Fully ordered system is not effectively complex. Sweet spot.

Chapter 11

Tensor States and Entangled Automata

Tensor Automata

The qubit automaton, elementary as it is, manifests definitive quantum phenomena in its agents. They include the primary role of observation, superposition of agent-states, quantum correlations of agents, and decoherence of quantum correlations by acts of interaction/observation. A solitary qubit, however, cannot exhibit a crucial quantum behavior: entanglement. For that, one requires at least two freedoms, two qubits bound together in a tensor product: the subject of this chapter.

The entanglement of quantum matter was brought forward in the twentieth century by Erwin Schrödinger. Though entanglement of quantum states is pervasively generated in macroscopic matter (in the complex interaction of particles in a gas or photons illuminating an object for example), entanglement of this sort is decohered rapidly as well. As the detailed trajectories of gas particles or photons is obscure to observation, so too is the entangling-disentangling dynamics of their interaction. To bring entanglement forward, one must consider simple systems of a few degrees of freedom with specially constructed Hamiltonians that evolve unentangled states into entangled states—as well as the inverse process. Such systems are typically not naturally occurring but are preeminent elements of *quantum computation* and other quantum technologies.

Like other important quantum processes, entanglement is observer dependent:

Principle (Entanglement is Observer Dependent). *The entanglement of a given agent depends on the observer. In particular, if an agent's state is self-observed to be an entangled state of high purity, observation by a complementary agent will find it dis-entangled and mixed.*

That the entanglement of a state is dependent upon its observation may seem surprising, but the Wheeler Principle and the modern discovery of the importance of *contextuality*[20] in quantum mechanics is consistent with this behavior. One can take some comfort in this surprise from the fact that self-observed agents hold a place of distinction. The self-observed Hamiltonian is an invariant of time-evolution providing stability and robustness to quantum evolutions even as other agents suffer the ravages and benefits of streams of spontaneous randomness.

Individual qubit agents must now be regarded as parts of a more comprehensive composite agent which has its own integrity. In particular, the state of being of an individual qubit agent in a tensor-product qubit network cannot generally be observed by other agents in the network. Qubit agent Individuality is generally lost to be succeeded by a new individuality at the tensor-product level. The qubit machines fashioning a composite machine in a multi-qubit network cannot generally be acted upon individually without doing violence to the whole. Such an observation of an entangled state is said to decohere it. The resulting observation is disentangled.

Entanglement

How does one describe the amount of entanglement of a quantum system? This is a complex topic in general but a robust procedure exists for the qubit pair—a bipartite system. This procedure is the Schmidt decomposition. Any bipartite tensor product state may be decomposed into its constituent eigenvectors and their eigenvalues and reassembled in *Schmidt form*[13, 12]. The amount of entanglement is measured by the number of non-vanishing Schmidt coefficients.

A second entanglement measure also exists for a bipartite system. This is the entanglement of formation and its measure is the entropy of the entangled agent observed by either of the entangling partners. The self-observed entropy of the entangled agent vanishes if the entangled state is pure. The entropy of the entangled agent as observed by either of the partners will be mixed.

The loss of qubit individuality in a tensor product of qubits makes it clear that one cannot speak generally about “one qubit observing the other.” Agents may only observe the whole entangled state.The observation of one qubit by another makes sense only if the qubits are in a product state, a special configuration in which parts can be separated without violating the whole.

Tensor product network agents thus fall into two classes: *separable or product* agents and *entangled* agents. Separable product agents can be factored from the tensor product and observed and acted on individually while preserving their information content. Entangled agents cannot because their information is distributed over the individuals. Entangled agents remain whole until they are acted upon by entanglement-breaking observation/interactions of other agents.

Complex systems of quantum agents in the computational representation are tensor products of qubits. Two freedoms are sufficient to portray entangled state dynamics. I'll call the tensor product network for a qubit pair the *qubit pair automaton*.

Qubit Pair Automaton

The qubit pair automaton is the next step up from the qubit automaton. With this step-up comes a capacity for entanglement. A pair of qubits ($N = 2, d = 4$) has $4N^2 - 1 = d^2 - 1 = 15$ physical agents consisting of $2N + 1 = d + 1 = 5$ species, each with $2N - 1 = d - 1 = 3$ varieties.,

$$\begin{pmatrix} \mathsf{Q}^{(1)} \otimes \mathsf{Q}^{(2)} & \mathsf{Q}^{(1)} \otimes \mathsf{P}^{(2)} & \mathsf{Q}^{(1)} \otimes \mathsf{I} & \mathsf{Q}^{(1)} \otimes \mathsf{S}^{(2)} \\ \mathsf{P}^{(1)} \otimes \mathsf{Q}^{(2)} & \mathsf{P}^{(1)} \otimes \mathsf{P}^{(2)} & \mathsf{P}^{(1)} \otimes \mathsf{I} & \mathsf{P}^{(1)} \otimes \mathsf{S}^{(2)} \\ \mathsf{I} \otimes \mathsf{Q}^{(2)} & \mathsf{I} \otimes \mathsf{P}^{(2)} & \mathsf{I} \otimes \mathsf{I} & \mathsf{I} \otimes \mathsf{S}^{(2)} \\ \mathsf{S}^{(1)} \otimes \mathsf{Q}^{(2)} & \mathsf{S}^{(1)} \otimes \mathsf{P}^{(2)} & \mathsf{S}^{(1)} \otimes \mathsf{I} & \mathsf{S}^{(1)} \otimes \mathsf{S}^{(2)} \end{pmatrix} \tag{11.1}$$

Note that each qubit identity I is present tensored with one of the physical agents ('logical' agent tensored with physical agent) and the tensor product of the identities of each qubit $\mathsf{I} \otimes \mathsf{I}$ becomes the sole identity of the pair. There is no need to distinguish $\mathsf{I}^{(2)} = \mathsf{I}^{(1)} = \mathsf{I}$, the identity on the 2-dimensional Hilbert space. Note also that $\mathsf{II} \equiv \mathsf{I} \otimes \mathsf{I}$ is the identity on the 4-dimensional tensor product space. Agents of the pair automaton are represented as 4×4 matrices and there are $d^2 - 1 = 15$ physical agents.

The number of agents appears to exponentially explode with the number of qubits N; but this is not true. Many of these additional agents are not independent. Each qubit contributes a pair of independent agents ('position-momentum') to the tensor product state:*

$$\{\mathsf{Q}^{(1)}, \mathsf{P}^{(1)}, \mathsf{I}^{(1)}, \mathsf{S}^{(1)}\}, \qquad \{\mathsf{Q}^{(2)}, \mathsf{P}^{(2)}, \mathsf{I}^{(2)}, \mathsf{S}^{(2)}\},$$

The maps from dependent to independent agents are matrix (dot) products of the agents of the canonical quartet. Consider the 6 products that involve the dependent agents $\mathsf{S}^{(1)}$ and $\mathsf{S}^{(2)}$. Consider first the products of $\mathsf{S}^{(1)}$. The agents $\mathsf{S}^{(1)} \otimes \mathsf{Q}^{(2)}$ and $\mathsf{Q}^{(1)} \otimes \mathsf{S}^{(2)}$ expressed in terms of the canonical agents are

$$\mathsf{S}^{(1)} \otimes \mathsf{Q}^{(2)} = (i\mathsf{P}^{(1)}\mathsf{Q}^{(1)}) \otimes \mathsf{Q}^{(2)}, \qquad \mathsf{Q}^{(1)} \otimes \mathsf{S}^{(2)} = i\mathsf{Q}^{(1)} \otimes (\mathsf{P}^{(2)}\mathsf{Q}^{(2)}).$$

while

$$\mathsf{S}^{(1)} \otimes \mathsf{S}^{(2)} = (i\mathsf{P}^{(1)}\mathsf{Q}^{(1)}) \otimes (i\mathsf{P}^{(2)}\mathsf{Q}^{(2)}).$$

*A systematic presentation of the agents of a general Nqubit system is presented in Appendix E.

Similar reasoning produces reductions of the remaining dependent agents leaving all agents accounted for in terms of the independent canonical quartet (11.3).

The notation can be simplified by hiding the tensor product symbol $\otimes$ with the understanding that, for the present discussion, adjacency of operators is to be understood as a tensor product of those operators, for example,

$$\mathsf{AB} \equiv \mathsf{A} \otimes \mathsf{B}.$$

Under this convention, a conventional matrix (dot) product will be indicated by context or, when necessary, explicitly as as $\mathsf{A} \cdot \mathsf{B}$.

Summarizing, the 15 physical operators of (11.1) make $d+1 = 5$ species each with $d-1 = 3$ varieties of agents. These may be arranged as five rows of species with three varieties in each row:

$$\begin{pmatrix} \mathsf{Q}^{(1)}\mathsf{I} & \mathsf{I}\mathsf{Q}^{(2)} & \mathsf{Q}^{(1)}\mathsf{Q}^{(2)} \\ \mathsf{P}^{(1)}\mathsf{I} & \mathsf{I}\mathsf{P}^{(2)} & \mathsf{P}^{(1)}\mathsf{P}^{(2)} \\ \mathsf{S}^{(1)}\mathsf{I} & \mathsf{I}\mathsf{S}^{(2)} & \mathsf{S}^{(1)}\mathsf{S}^{(2)} \\ \mathsf{P}^{(1)}\mathsf{S}^{(2)} & \mathsf{Q}^{(1)}\mathsf{P}^{(2)} & \mathsf{S}^{(1)}\mathsf{Q}^{(2)} \\ \mathsf{S}^{(1)}\mathsf{P}^{(2)} & \mathsf{Q}^{(1)}\mathsf{S}^{(2)} & \mathsf{P}^{(1)}\mathsf{Q}^{(2)} \end{pmatrix} \tag{11.2}$$

Each of these agent-actors is fixed by the specification of the four independent agent-actors of the canonical quartet.

Consider now the independent and dependent agents. Following the principle that each degree of freedom supports two independent agents, a canonical position and momentum, the two-degree of freedom pair automaton has four independent agents. Take as this independent set the canonical pair for each qubit tensored with the identity on the other qubit: a canonical "quartet."

$$\mathsf{Q}^{(1)}\mathsf{I}, \qquad \mathsf{I}\mathsf{Q}^{(2)}, \qquad \mathsf{P}^{(1)}\mathsf{I}, \qquad \mathsf{I}\mathsf{P}^{(2)}. \tag{11.3}$$

The maps from dependent to independent agents are matrix (dot) products of the agents of the canonical quartet. For example,

$$\mathsf{Q}^{(1)}\mathsf{Q}^{(2)} = \mathsf{Q}^{(1)}\mathsf{I} \cdot \mathsf{I}\mathsf{Q}^{(2)}, \qquad \mathsf{Q}^{(1)}\mathsf{P}^{(2)} = \mathsf{Q}^{(1)}\mathsf{I} \cdot \mathsf{I}\mathsf{P}^{(2)}.$$

A similar expression of the product agents

$$\mathsf{P}^{(1)}\mathsf{Q}^{(2)}, \quad \mathsf{P}^{(1)}\mathsf{P}^{(2)}.$$

exists in terms of the canonical quartet. All dependent agents are now accounted for in terms of the independent canonical quartet (11.3).

Each row of these arrays is a set of 3 varieties of a given species and there are 5 species. The varieties in each row commute; the species in each column do not.

Three of the species (the upper three rows) consist of product varieties. When formed into superpositions to create other agents, they may be decomposed into their single qubit parts. The two bottom rows consist of varieties that when superposed to create other agents cannot be decomposed: these are entangled qubit pair agents. Their state of being and action cannot be isolated into parts but is shared: a single whole of the agent built from them.

As a sample of the structure of tensor product agents, consider the agent-actors $\mathsf{P}^{(1)}\mathsf{S}^{(2)}$ and $\mathsf{Q}^{(1)}\mathsf{P}^{(2)}$ in either of the bottom two rows of the array of agents (11.2). These are varieties of the same species so they commute.

$$[\mathsf{P}^{(1)}\mathsf{S}^{(2)}, \mathsf{Q}^{(1)}\mathsf{P}^{(2)}] = 0, \qquad [\mathsf{P}^{(1)}\mathsf{S}^{(2)}, \mathsf{Q}^{(1)}\mathsf{P}^{(2)}] = 0.$$

Autonomous Tensor Chemistry

The tensor product of a pair of agents

$$\mathsf{a} = \sum_j |\mathsf{a}, j\rangle \mathfrak{p}_{\mathsf{a}j} \langle \mathsf{a}, j|, \qquad \mathsf{b} = \sum_k |\mathsf{b}, k\rangle \mathfrak{p}_{\mathsf{b}k} \langle \mathsf{b}, k|$$

is

$$\mathsf{ab} = \frac{1}{2} \sum_{j,k} a_j^* a_k b_j^* b_k \ |\mathsf{a}, j\rangle\langle \mathsf{a}, k| \otimes |\mathsf{b}, j\rangle\langle \mathsf{b}, k|.$$

As introduced above, the tensor product $\mathsf{a} \otimes \mathsf{b}$ is indicated by adjacency ab, where $|\mathsf{a}, j\rangle\langle \mathsf{a}, j|$ labels the states of a and similarly for b. The partial trace over b of the tensor product ab, ($Tr_{\mathsf{b}}\mathsf{ab}$) projects the tensor product onto agent-states of a (tracing over thos of b. It is thus an operator with states $|\mathsf{a}_j\rangle\langle \mathsf{a}_k|$ and matrix elements $\langle \mathsf{b}_j|\mathsf{b}_k\rangle$ that are the amplitudes of those states,

$$Tr_{\mathsf{b}}\mathsf{ab} = \frac{1}{d} \sum_{jk} |\mathsf{a}, j\rangle\langle \mathsf{b}, j|\mathsf{b}, k\rangle\langle \mathsf{a}, k|.$$

The building-up of tensor product agents from elementary factors is software chemistry in which tensor product atoms are combined into tensor product molecules. Consider tensor product agent-states corresponding to the agent-actors (11.2)

$$\begin{pmatrix} \mathsf{q}^{(1)}\mathbf{i} & \mathbf{i}\mathsf{q}^{(2)} & \mathsf{q}^{(1)}\mathsf{q}^{(2)} \\ \mathsf{p}^{(1)}\mathbf{i} & \mathbf{i}\mathsf{p}^{(2)} & \mathsf{p}^{(1)}\mathsf{p}^{(2)} \\ \mathsf{s}^{(1)}\mathbf{i} & \mathbf{i}\mathsf{s}^{(2)} & \mathsf{s}^{(1)}\mathsf{s}^{(2)} \\ \mathsf{p}^{(1)}\mathsf{s}^{(2)} & \mathsf{q}^{(1)}\mathsf{p}^{(2)} & \mathsf{s}^{(1)}\mathsf{q}^{(2)} \\ \mathsf{s}^{(1)}\mathsf{p}^{(2)} & \mathsf{q}^{(1)}\mathsf{s}^{(2)} & \mathsf{p}^{(1)}\mathsf{q}^{(2)} \end{pmatrix}, \tag{11.4}$$

where $\mathbf{i} = \frac{1}{2}\mathsf{I}$ is the fully random state. Because its factors differ only by the fully random state which is proportional the identity, one readily sees that if an agent-actor in (11.2) participates in an entangled state, the corresponding agent-state will be entangled.

Arrays (11.2) and (11.4) hold the elementary components of a qubit pair automaton, only four of which (such as those in the upper left corner) are independent. Each of the independent agent-states holds 1 bit of information and the four together hold the 4 independent bits of information of the qubit pair.

Canonical Coordinates: Tensor Atoms

The four canonical agents paired with the fully random state (the normed identity) in the upper left-hand corner of (11.4) form a 'canonical agent-state quartet.' I shall call them *tensor atoms*. The quartet of tensor atoms for the qubit pair are an independent set which fix the values of dependent states.

$$\mathsf{q}^{(1)}\mathbf{i}, \qquad \mathbf{i}\mathsf{q}^{(2)}, \qquad \mathsf{p}^{(1)}\mathbf{i}, \qquad \mathbf{i}\mathsf{p}^{(2)}. \tag{11.5}$$

A tensor atom is *atomic* in an informational sense: it contains but one bit of definite information. Only *one* agent-state's information appears in a bipartite tensor product as that definite bit. The tensor atom is, nonetheless, *bi-partite*, a tensor product of *two* qubits, the second qubit being that of the random agent **i**.

A tensor atom acting as observer can observe one bit of a tensor product state, thereby decohering it, while leaving the other bit undisturbed. We shall see this important property of *selective* observation in action in Chapter 10.

Since the information of only one agent-state is present in a bipartite tensor atom, the atom can be named for that agent. For example, the tensor atom $\mathsf{q}\mathbf{i}$ of a position agent q can be called a 'position atom.' There is, however, a second position atom, $\mathbf{i}\mathsf{q}$. Tensor atoms are seen to be *polarized* left and right. The canonical quartet of tensor atoms can therefore be classified as:

$\mathsf{q}\mathbf{i}$ Left Position,

$\mathbf{i}\mathsf{q}$ Right Position,

$\mathsf{p}\mathbf{i}$ Left Momentum,

$\mathbf{i}\mathsf{p}$ Right Momentum.

The identity exists because of the requirement that the identity complete the qubit's algebra. Manifest in an agent-state, the identity tells us that a random element is an intrinsic element of the qubit automaton's algebra. A tensor atom is impure with an entropy of 1 bit—the one impure agent-state among a remainder of pure agent-states $\mathsf{q}, \mathsf{p}, \mathsf{s}$. Moreover, the identity makes tensor atoms possible. Tensor atoms have a key role to play in observation.

A canonical quartet that describes the bipartite world of tensor atoms in the Pauli state (4.11) is

$$\mathsf{q}\mathbf{i} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{i}\mathsf{q} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$\mathsf{p}\mathbf{i} = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}, \quad \mathbf{i}\mathsf{p} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix},$$

Dependent agent-states are algebraic functions of the canonical quartet. Just as the 11 dependent tensor product agent-actors may be represented in terms of a canonical quartet in the foregoing discussion, so too may the agent-states be represented in a canonical quartet of states (11.4). The maps from independent agent-states to dependent ones are presented in Appendix B.

Tensor Molecules

A pair of Atomic agents may be bound by their inner product to create bipartite tensor molecules,

$$\mathsf{q}^{(1)}\mathbf{i} \cdot \mathbf{i}\mathsf{q}^{(2)} = \mathsf{q}^{(1)}\mathsf{q}^{(2)}, \qquad \mathsf{p}^{(1)}\mathbf{i} \cdot \mathbf{i}\mathsf{p}^{(2)} = \mathsf{p}^{(1)}\mathsf{p}^{(2)}, \qquad \mathsf{s}^{(1)}\mathbf{i} \cdot \mathbf{i}\mathsf{s}^{(2)} = \mathsf{s}^{(1)}\mathsf{s}^{(2)},$$

The inner product of oppositely polarized tensor atoms has the remarkable property of annihilating the random agents in the formation of the tensor molecule bond. The entropy of the pair is reduced to zero. A tensor molecule formed from impure tensor atoms is pure with vanishing entropy as in the above illustrations!

Tensor (product) atoms are the elementary building blocks of tensor (product) molecules. They create an information or software-based ’chemistry’ originating in the tensor product bond. I am using these terms for the computational software representation of these agents which are in the physical chemistry hardware of the force field. I do not imply a direct connection of ‘mathematical’ tensor atoms and molecules to ‘hardware’ physical atoms and molecules which underlie their structure. Nonetheless, I expect there are significant morphisms between the hardware chemistry of physical atoms and molecules, particularly in condensed matter, and the software or computational chemistry of tensor atoms and molecules.

Selective Observation and Quantum Disturbance

A tensor atom, qua agent, has the power to selectively observe a particular qubit in a tensor product (thereby decohering it in the observer's frame of reference), without disturbing others. This is an important effect.

Principle (Selective Observation)**.** *A tensor atom agent-state, acting as an observer of tensor product agent-states (not necessarily atoms), selectively observes the qubit in the tensor product associated to its atom, thereby decohering it. The remaining qubits are undisturbed.*

In Chapter 3, in the section,"The Random Agent-State and Information Acquisition," I pointed out that an agent entering a random state carries away no information from the agents with which it is interacting and which evolved it to that state. In Chapter 6, in the section, "Information Flow in Automata" I described how information is lost in the reference frame of the observer due to the decohering dynamics of motion—an illustration of the fundamental quantum principle that the act of observation disturbs the quantum agent observed in the reference frame of the observer.

The paradigm of information loss by agent observations is by the complementary pair (Chapter 5) which are the most elementary representatives of quantum species. A position agent q gains no information whatsoever in observing its complement, the momentum agent p, and vice-versa. The dynamic in which no information is found out by agents q and p observing one another is a transition to the fully random state by the observing agent:

$$(\mathsf{q} \parallel \mathsf{p}) = \mathbf{i}, \qquad (\mathsf{p} \parallel \mathsf{q}) = \mathbf{i},$$

The existence of tensor agent-states binding together multiple qubits opens up the process of observation and its attendant transfer of information by observing agents to be *selective*. Observation can be selectively applied to individual qubits of a tensor product. An agent can gain *some* information about a tensor agent-state without obtaining it all. The information of the qubit observed is destroyed from the perspective of the observing agent and replaced by completely random information; the qubits not observed remain undisturbed.

Selective observation is implemented by a tensor atom agent of the same species as the agent whose bit is to be observed. Here is an illustration. A selective observation is to be made, say on the left bit of an agent in the tensor product state qp. A left position atom of the same species $\mathsf{q}'\mathbf{i}$ can do the job. The result is a faithful observation of the left bit and a random observation of the right bit

$$(\mathsf{qp} \parallel \mathsf{q}'\mathbf{i}) = \mathsf{q}\mathbf{i}.$$

On the other hand, if the observing tensor atom has the momentum agent holding the left bit, $\mathsf{p}'\mathbf{i}$, the results are as one expects: the momentum agent of the left observing bit acting on the position of the first observed bit receives a fully random state:

$$(\mathsf{qp}\ \|\ \mathsf{p}'\mathbf{i}) = \mathbf{ii}.$$

The results are similar if the observed and observing atom are of the same species: faithful observation of the right bit in this case,

$$(\mathsf{qp}\ \|\ \mathbf{i}\mathsf{p}) = \mathbf{i}\mathsf{p}.$$

But if the observing atom is of a different species from the bit observed, the fully random state results,

$$(\mathsf{qp}\ \|\ \mathbf{i}\mathsf{q}) = \mathbf{ii}.$$

Although q and p are complementary agents which cannot observe one another with definite results, they can observe and be observed in a tensor product in which the complementary bits are separately observed by an observing atom of the same species, as above.

When applied in observation to a tensor product agent, the random bit of the atom acting on the observed agent, carries away no information; it leaves the observed bit unchanged. The observation of the definite bit, on the other hand, depends on the atom. If the atom is the same species as the observed bit, a definite result is obtained. If the atom is of a different species, a random result is obtained. In that case, the observing agent carries away *no information* from the observed agent, none from the right or left bit.

A tensor atom or molecule—as an observing agent —must be of the same species as that of the agent whose bit is to be observed if it is to be observed faithfully. For example, if the left bit of a tensor product agent, such as qp is to be selectively observed, the tensor atom acting as observer must be a left position atom of the same species, position in this case. Similarly, if the right bit above is to be observed, the observing atom must be of the same species—a right momentum atom.

One can readily see that what holds true for a tensor atom (for which one of the bits is associated to the random agent-state) also holds for a tensor molecule selectively observing one of the bits of another molecule. That bit must be associated to an observing bit of the same species for information gain..

The agent associated to the random observing bit of an atom carries away no information. The qubit it acts on will remain undisturbed. If both bits of a tensor molecule are of a disparate species relative to an a observing molecule, the result is a completely random state on both bits. No information whatsoever can be gained by the observing agent-state—as a tensor molecule of disparate species relative to that observed.

$$(\mathsf{ab} \parallel \mathsf{q}\mathbf{i}) = (\mathsf{a} \parallel \mathsf{q})\mathbf{i}.$$

The observation of a pair of entangled agents is not the same as the tensor product of the observed factors. An entangled agent-state, such as $(\mathsf{qp} + \mathsf{pq})/2$ is not the same is the observed entanglement by some agent a—observation does not distribute over tensor products of agent-actors or agent-states,

$$(\mathsf{qp} + \mathsf{pq} \parallel \mathsf{a}) \neq (\mathsf{q} \parallel \mathsf{a})(\mathsf{p} \parallel \mathsf{a}) + (\mathsf{p} \parallel \mathsf{a})(\mathsf{q} \parallel \mathsf{a}).$$

The observational status of agents for multiple qubits is richer than that for the isolated qubit. Agents aggregate into species and varieties. Faithful observation holds between agents of the same species. Agents from different species cannot faitfully observe one another. In the modern quantum literature this is known as *quantum contextuality* [20]. Varieties of the same species share the same context.

Automata Observation and Partial Trace

Composite quantum systems existing as entangled states are separated in quantum computation by the partial trace operation. The result of this operation is a mixed state density matrix reduced in size by the size of the of subsystem traced over. Notably, if the original self-observed density matrix was entangled and pure, its observation by an agent of a different species is disentangled and maximally mixed.

Partial trace is a quantum decohering operation of human design and implementation, notably in quantum computation. It's possible that such operations can be found in the chemistry of nature outside of human agency, but examples are unknown as of this writing.

While partial trace is a quantum technology operation of what might be called 'artificial physics,' decoherence through agent observations is a natural operation not peculiar to human technology. Quantum agents reproduce the results of partial trace within a much more general repertoire of decohering-disentangling operations that are natural byproducts of agent observations. One can describe decoherence of entanglements as a natural phenomenon in complex matter made possible by the abundance of agents in dense matter. These agents serve the mutual roles of quantum random observations of the Hamiltonian and the chaotic amplification of these randomly observed Hamiltonian directives through the unstable collision manifolds of dense matter such as gases, liquids.

Partial trace is closely related to observation by tensor atoms. A tensor atom observer which will produce a partial trace accepts any state for the qubit to be traced over but holds the qubit state (as observer) of the qubit not traced over.

For example, if the second qubit representing the agent b in the product ab is to be traced over, this is accomplished by observation by a tensor atom of the first, namely, $\mathsf{a}\mathbf{i}$:

$$(\mathsf{ab} \parallel \mathsf{a}\mathbf{i}) = \sum_j |\mathsf{a}\mathbf{i}, j\rangle \mathfrak{p}_{\mathsf{a}\mathbf{i}j|\mathsf{ib}} \langle \mathsf{a}\mathbf{i}, j|.$$

where the probability of the state $|\mathsf{a}\mathbf{i}, j\rangle\langle \mathsf{a}\mathbf{i}, j|$ emerging in an observation of ab is

$$\mathfrak{p}_{\mathsf{a}\mathbf{i}j|\mathsf{ab}} = \langle \mathsf{a}\mathbf{i}, j|\mathsf{ab}|\mathsf{a}\mathbf{i}, j\rangle = \mathsf{Tr}\left(|\mathsf{a}\mathbf{i}, j\rangle\langle \mathsf{a}\mathbf{i}, j|\, \mathsf{ab}\right).$$

Illustration: Bell States

The process may be illustrated with exemplary bipartite states—the Bell states—that form a complete basis for a qubit pair and are completely entangled. In observing these states within the quantum automata formalism, regard them as Hamiltonians

$$\mathsf{h}_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}, \quad \mathsf{h}_2 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\mathsf{h}_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathsf{h}_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

The partial trace of these states with respect to either qubit produces a completely mixed state in the smaller, reduced two-dimensional Hilbert space.

A tensor product atom (11.5) serves as an observer or "probing agent" which can be taken as a Pauli state (4.3) on the first qubit, $\mathsf{q}\mathbf{i} \equiv \hat{\mathsf{q}}\mathbf{i}$. When observed by the probe according to the Observation Postulate III, the Bell states yield a fully mixed pair of states, one from each of the qubits, superposed with a pair of null states:

$$(\mathsf{h}_1 \parallel \mathsf{q}\mathbf{i}) = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}, \quad (\mathsf{h}_2 \parallel \mathsf{q}\mathbf{i}) = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix},$$

$$(\mathsf{h}_3 \parallel \mathsf{q}\mathbf{i}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (\mathsf{h}_4 \parallel \mathsf{q}\mathbf{i}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Eliminating the null states of a tensor product observed by a tensor atom yields a reduced density matrix from above equivalent to that obtained by partial trace over the bit .

Observation is a richer disentangling operation than partial trace. Like partial trace, observation decoheres entangled sates to mixed states—but in concert with null states. The observed states are partly decohered but partly entangled. One of the states in the mixture is from the first qubit and the other is from the second. Each is padded with a null state. These states are both entangled and mixed with Schmidt number 2 whereas the Bell state from which they were observed was fully entangled with full Schmidt rank of 4.

Each of the independent agents holds 1 bit of information. Individual qubit agents that compose an entangled qubit pair do not generally have the power to observe one another. Only tensor product agents have the power of observation—and it is the power to observe one another in their wholeness as tensor products generally. However if the tensor product agent is in a product state, the individual agent factors that compose the product gain the observing power of their individuality.

Illustration: Free Motion

Observations of agents in in Pauli states under the little Heisenberg Hamiltonian are likewise mixed states. The position and and action agent observations of the Hamiltonian (6.4) are

$$(\mathsf{h} \parallel \mathsf{q}) = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}, \qquad (\mathsf{h} \parallel \mathsf{p}) = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{2} \end{pmatrix} \qquad (\mathsf{h} \parallel \mathsf{s}) = \begin{pmatrix} \frac{1}{2} & \frac{1}{6}i \\ -\frac{1}{6}i & \frac{1}{2} \end{pmatrix},$$

In any instance of interaction the instantaneous observed Hamiltonian will be one or the other of $\mathsf{h}_1, \mathsf{h}_2$, but it is not definite (not repeatable under fixed conditions). The hamiltonian self-observed is, of course, a temporal invariant.

Evolution under the little Heisenberg Hamiltonian is stasis. The equation of motion for any of the agents, exemplified by q, is the statistical stationary state

$$\frac{d\mathsf{q}}{dt} = (i/\hbar)[\mathsf{q}, (\mathsf{H})_{\mathsf{q}}] = \mathbf{0}, \qquad \mathsf{q}(t) = const,$$

and similarly for the other agents. These agents hold their Pauli states (4.11) in self-observation, but their observation in the Hamiltonian by other complementary agents contain random data, an example of the Peres Observation Principle.

Emergent New Information

If agents were omniscient and had complete access to one another's information, the observation of agent a's state of being by agent b would be identical to a's dynamic state. This is not the case in automata theory because agent b only has access to agent a's state within the community of states registered in the Hamiltonian. This accounts for the fundamental differences between information acquisition and transmission in quantum systems compared to classical.

The act of observation introduces new information into a quantum system because some of the observed signal from the Hamiltonian by each agent may be random. Random information is new information; it was not produced by arithmetically processing previous state information (von Neumann's sin). Since the agent acts on that information dynamically, the randomly acquired information has dynamic consequences. It becomes entrained in the evolutionary flow of quantum motion. Random data appearing in agent observations is the source of new information flow into nature.

Principle (Emergent Information). *New information enters a quantum system brought in by agent evolution directed by an observed Hamiltonian with some random data.*

Abundant Randomness

Readers must be struck by the copious amounts of randomness produced through quantum observation in the foregoing illustrations. The reason for this is this illustration is for an isolated single degree of freedom. An isolated qubit automaton is, roughly speaking, "half-random, half-definite" and the universe of an isolated qubit automaton a "half-random, half-definite" universe.

My narrative thus far without further elaboration is most naturally applicable to a hot, dense gas in which otherwise structureless particles travel over free paths between collisions as force fields act: a gas of qubit automata. Such a gas is indeed beset with a high degree of randomness. Unstable collision manifolds magnify the quantum randomness of agent interaction/observations.

To go beyond the universe of the qubit one must take into account the way degrees of freedom reestablish themselves in different phases of matter and the nature of the force fields that sustain them. The formation of condensed matter and the containment of the far-ranging translational degrees of freedom in a gas within the far more restricted amplitudes of roton and vibratory modes in condensed matter is a critical process in the creation of order in the universe (Chapter 8). Chemical bonding further sets natural "containing structures" in place to bring further order into matter.

Time Scales

The time scales for observation and execution of motion are set by the physics of the Hamiltonian, in particular, the energy levels of the Hamiltonian. The Hamiltonian of a complex system may contain many energy levels and time scales. Let us focus on a single energy scale $|\mathsf{H}|$. The characteristic time for the process represented by that Hamiltonian is

$$\tau \approx \hbar/|\mathsf{H}|.$$

Some typical examples are,

$$\text{For molecular bond vibrations} \qquad \hbar/|\mathsf{H}| \approx 10^{-15} sec.$$

$$\text{For flagella rotations} \qquad \hbar/|\mathsf{H}| \approx 10^{-1} - 10^{-2} sec.$$

$$\text{Motor protein walking} \qquad \hbar/|\mathsf{H}| \approx 1 sec.$$

Observation as Motion: Reflections

Quantum agents achieve their identity dynamically in the act of motion. An agent becomes "what it is" by observing the Hamiltonian superagent. The act of observation—interaction—triggers the Hamiltonian into action through the time-evolution operator to produce the agent's new identity as its evolved state in time.

We can say, "agents come to know themselves in observing, not themselves, but in observing the community of agents manifested in the Hamiltonian superagent." That agent-observed Hamiltonian superagent then acts dynamically creating the agent's new state in time. The process might better be described as an agent's "self-creation" or "self-evolution" rather than self-observation. Of course, it is not the isolated agent that creates itself. It is created by the community of agents embodied in the Hamiltonian evolving its previous state to the new state. The process of self-observation is intrinsically social. An agent comes to know itself in observing the community of agents in the Hamiltonian.

Throughout nature, perceiving and receiving data and acting on it is indigenous. Animals, not just human beings, sense their environments and act in response. Observations induce change in behavior. Within the cell, numerous agents from motor proteins to molecules and ions in chemical circuits and cycles transmit and receive data in binding and releasing ions and proteins, and other chemical species which induce actions in these microscopic machines. In observing other agents and acting on the basis of those observations quantum agents present an elementary paradigm.

In these activities, agents with finite capacities act with incomplete data which is filled out with random data as assumptions or guesses in human beings and highly evolved animals. Action is executed routinely in nature by agents with incomplete information about the environments in which they must act. Incomplete information on the classical level is a consequence of the limited memory capacity and processing ability of macroscopic agents: most agents have far less capacity than would be necessary to hold and process all the information available to them.

At the microscopic quantum level, incompleteness of agent information is more than a limitation of information processing capacity; it is a consequence of the limitation of definite information exchange between complementary agents: *Heisenberg uncertainty.* This is a law of nature. Quantum agents act in executing their dynamics with some random information. This random information is intrinsic to nature.

A Coding Theorem

The encoding of information in computers is a familiar operation. A hardware register of some construction based on a physical effect (transistor semiconductor levels, atomic spins, etc.) holds the information. Let us say the bits in a particular hardware register are set to encode the letter "*A*" in some code, say an 8 bit register in ASCII code. When the bits in the register are set to 01000001, they encode the letter "*A*." But these bits also represent a state of nature, a set of atoms in a containing structure with its electronic states or magnetic spins in a certain state. This is the "natural" state of the physical matter independent of our regarding them as code for a letter of the alphabet. Yet this same set of electronic levels or spin states also represents the letter state *A* according to the encoding design of the register. How can the same register encode two quite different states, say transistor levels or physical spins on the one hand, and the letter "*A*" on the other? How does nature manage this dual representation?

The answer is in the dynamics that set the register according to a Hamiltonian designed to encode the "*A*". The particular design Hamiltonian, for example, acts on a register in a standard state (say all bits set to "0") and dynamically advances them to the bit pattern 01000001 for "*A*". The dynamic of motion, the action of the Hamiltonian in advancing the states from the standard state to the encoded "*A*" has set up a correspondence between the physical states of the transistor or spins and the code of the letter "*A*". Dynamic processes of nature yoked to the design of a computer register bring forth the coincidence of a particular set of physical spins and the letter "*A*".

This process is fundamental to nature's capacity for agents to form *representations* of other agent-states in their own physical states. The Hamiltonian that executes this process is the critical part of the encoding.

Summary

The theory of quantum automata rests on individual canonical agents ("positions and momenta"), a canonical pair for each degree of freedom of the underlying force field. Agents observe and gain information about other agents, not individually, but in concert in their superposed presence in the Hamiltonian—a global superagent. Agent observed Hamiltonians are postulated to follow well-established quantum protocols of the Born Rule. This means agents hold Hamiltonian observations that may differ from one another and these differences may be random.

The time-evolution operator acting on each agent is postulated to be the time-evolution operator directed by the Hamiltonian observed by that agent. In this manner, a stream of random information enters quantum dynamics. An important instance of this postulate is the Hamiltonian operator-agent itself whose evolution is directed by its self-observation. The Hamiltonian self-observation is faithful and evolves unitarily without random input though this is not necessarily the case for other agents.

Quantum agency embodies both subjective and objective traits. An agent's state is stored in its own register—a clear instance of subjectivity. The states of objects (states of other agents) are stored as the state of the community of agents in the Hamiltonian's registers—a superposition of all the agent-states, a quantum 'objectivity.'

Subjects learn about objects as the observed state of the community of agents. Objectivity is a communal concept. Objectivity arises from the presence of multiple individual agent perspectives in the Hamiltonian superposition. All agents have a voice in the object that codes their evolution—the Hamiltonian, the superagent representing the community of other agents. "Subjects learn about other agents" in the scenario above by observing the Hamiltonian. Their observation is integral to and occurs in the mechanics of motion. The result of their observation is revealed in the next state to which the time evolution operator (executing their observed Hamiltonian) moves them.

Quantum agents are ceaselessly acting, observing and moving according to the results of observation in a universe in which time is unstoppable.

Chapter 12

Entropy and Automata

Entropy is a measure of the indefinite information held by an agent. Since quantum agents acquire information about other agents in the course of their motion by their observation of the community of agents represented by the Hamiltonian, entropy is a measure of an agent's uncertainty about the state of the community of agents represented by the Hamiltonian. Thus, entropy in Autonomous Quantum Theory is not a single unique quantity. Entropy is agent-specific. The Hamiltonian agent is noteworthy in this regard.

The Hamiltonian superagent's self-observation for faithful observation had a vanishing entropy but this will not be the case for the common entropy of the communal observation.

A typical agent, such as q, observes the Hamiltonian (receives the Hamiltonian state h) as an excitation of its registers $\mathsf{q}_j, j = 1, 2$ with probabilities

$$\mathfrak{p}_{\mathsf{h}|\mathsf{q}j} = \langle \mathsf{q}, j|\mathsf{h}|\mathsf{q}, j\rangle = \mathsf{Tr}\left(|\mathsf{q}, j\rangle\langle \mathsf{q}, j|\,\mathsf{h}\right). \tag{12.1}$$

The number of bits required to represent a probability $\mathfrak{p}_{\mathsf{h}|\mathsf{q}j}$ is $-\log(\mathfrak{p}_{\mathsf{h},\mathsf{q}j})$. The uncertainty measured in bits of the communal state h by agent q is then the sum of these bits weighed by the probability of each:

$$\mathsf{S}_{\mathsf{h}|\mathsf{q}} = -\sum_j \mathfrak{p}_{\mathsf{hq}j} \log(\mathfrak{p}_{\mathsf{h},\mathsf{q}j}). \tag{12.2}$$

To recapitulate, this is the the entropy experienced by agent q in observing the community of agents h.

Since entropy is local to each observing agent, the assignment of an entropy to the entire system of agents requires further assumptions typical of the assumptions made in Statistical Mechanics, such as all microstates of a system of automata are equally probable subject to macroscopic constraints.

Tensor Atom Entropy

Quantum automata possess an intrinsic entropy in their tensor structure. Tensor atoms (11.5) hold 1 bit of random information. This random information is annihilated when a right tensor atom forms a tensor molecule with a left tensor atom under the inner product operation. (Chapter 7). A self-observed tensor atom is impure. The tensor molecule formed from tensor atoms can be pure!

Chapter 13

Paradoxes, Double-Slits, and Erasers

> *... the double slit experiment is a phenomenon which has in it the heart of quantum mechanics; in reality it contains the only mystery of the theory.*
>
> R. P. Feynman [24]

This chapter takes up the most challenging and puzzling phenomena in Quantum Mechanics from the perspective of Autonomous Quantum Theory. Our guide will be the Principles of Observation articulated by J. A. Wheeler and Asher Peres (Chapter 3). I summarize and comment on them briefly:

Principle (Quantum Observation)**.** *(i.) Quantum Observables are also Observers. As observers and observables, they are agents with states of being and capacity for action to change one another.*

(ii.) Quantum agents come in species (Chapter 3). Members of the same species faithfully observe and are observed by one another. Members of different species receive random signals when they attempt to observe one another. The fundamental agents of each degree of freedom—a canonical position and momentum—are different species and cannot faithfully observe one another.

(iii.) Tensors bind elementary agents "tensor atoms" together in tensor product agents. Elementary agents are factors of the tensor product and may be of various species. Tensored agents, acting as observers, may selectively observe individual agent factors faithfully if they coincide in the order of the factors and are of the same species of the observing factor; otherwise, they receive a random signal as their observation.

In addition I shall rely on the principle of *Superposition and Context* (Chapter 3) in which a superposition of canonical agent-states in one context, such as position, transforms them into the agent-states of the complementary context, such as momentum.

The Phenomena

Behold the moon. Now cover your eyes. Did it cease to exist? Uncover your eyes. There is the moon just as you expect. Check more carefully. Your friend, standing at your side, eyes uncovered while you covered yours, reports the moon never ceased to be. Is any of this weird? Not at all.

Losing sight of things because of blockage, poor light, or any other number of impediments that temporarily destroy vision is not weird. This is not what makes quantum observation weird. Einstein knew that when he asked the question, "Is the moon there when nobody looks?" He had good reason to ask.

Replace the stream of moonlight by one or two of its photons. Replace your eyes with charge-coupled-devices (CCDs) capable of recording single photon impacts. Enter the quantum world of one, two, or just a few quanta. Einstein's question is to the point in this world.

Photons from the moon entering your eye carry pieces of information from a multitude of patches of the moon's surface. They form its image mutually decohering one another into a macro-image averaged and blended over all its quantum surface fluctuations—on receivers that range from from the retina of the eye to CCD detectors.. If a photon or two were scattered off, and lost to you, you wouldn't notice. But at the nanoscale, the observation event involves only one or two photons that can sail along in superposition or entanglement before colliding with another and decohering. If one were lost, the observation event would be irreparably damaged.

These "weird" occurrences are perfectly normal under the Peres and Wheeler observation principles which come into their own at the nanoscale of the quantum. These paradoxical behaviors are statistical: they rely not on a single test but statistics from a multitude of tests.

Quantum observation seems weird because"looking" (what will be observed) depends on the state of the object "looked at." That seems normal enough. But in nanoscale observations where peculiarly quantum effects enter, the state of the "looker" is equally important and the The Peres Observation Principle enters. The state of the observing agent plays a determinate role in what that agent can observe because observer and observed must be of the same species, that is, share the same quantum context (Chapter 3) if they are to faithfully communicate. These peculiarly quantum effects are unnoticeable in ordinary life immersed in a world of abundant quanta with photons from the moon or any other macro-scale object and trillions upon trillions of atoms and molecules with which to receive, observe, and decohere them.

Observers can experience an interference pattern from the quanta emanating from a pair of slits if they give up knowing which slit each quantum passed through. This is because the state of a particle of interest, called the 'signal," and the state of a particle acting as a detector, called the "indicator," are entangled as I shall shortly explain. Interference patterns appear when the species (quantum context) of the observing agent is the same as that of the agent observed and disappear when they differ. This is not weird, but a natural, even marvelous, spectacle.

The discussion that will follow is in a in a modern human, rather than completely natural, context for the most part. The quantum agents involved are products of quantum technology involving Lasers, SPDC crystals, CCD detectors, Beamsplitters, Mirrors, and Coincidence Counters.

Double-Slit "Superposer"

How does one create a quantum superposition? A beam of quanta incident on a double slit readily does the job. The two beams exiting the slit are in superposition as long as they are not disturbed. The double-slit is a paradigmatic device for creating quantum superpositions.

The Double Slit and its Agents

The continuous beam of particles passing through the double slit can be represented as a qubit where each of its two states is a continuous function of a position coordinate, one for the path through each slit, x_1 and x_2 rather than a discrete number as in previous discussions.

Represent the particle-beam's agent-states as position agents. This is a qubit automaton whose position agent-states are functions $|\mathsf{q},1\rangle(x_1)$ passing through the first slit and and $|\mathsf{q},2\rangle(x_2)$ passing through the second. If the particle-beams are plane waves with wave number k and frequency ω in time t, the pure position state-vectors are

$$|\mathsf{q},1\rangle(x) = e^{i(kx_1-\omega t)}\begin{pmatrix}1\\0\end{pmatrix}, \qquad |\mathsf{q},2\rangle(x) = e^{i(kx_2-\omega t)}\begin{pmatrix}0\\1\end{pmatrix}.$$

These beam states are orthogonal and complete

$$\langle \mathsf{q}, j|\mathsf{q}, k\rangle = \delta_{jk}, \qquad \sum_j |\mathsf{q}, j\rangle\langle \mathsf{q}, j| = \mathsf{I}.$$

If only one or the other slit is open, the agent-state density matrices at the exit of the slits are respectively

$$\begin{aligned} \mathsf{q}_1(x_1) &\equiv |\mathsf{q}, 1\rangle(x_1^*)\langle \mathsf{q}, 1|(x_1) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \\ \mathsf{q}_2(x_2) &\equiv |\mathsf{q}, 2\rangle(x_2^*)\langle \mathsf{q}, 2|(x_2) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned} \tag{13.1}$$

When both slits are open, the state-vector at the exit of the slits is a superposition of two independent combinations of beams, which can be taken as sum and difference*

$$\begin{aligned} |+\rangle &= \frac{1}{\sqrt{2}}(|\mathsf{q}, 1\rangle(x_1) + |\mathsf{q}, 2\rangle(x_2)) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{ikx_1} \\ e^{ikx_2} \end{pmatrix}, \\ |-\rangle &= \frac{1}{\sqrt{2}}(|\mathsf{q}, 1\rangle(x_1) - |\mathsf{q}, 2\rangle(x_2)) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{ikx_1} \\ -e^{ikx_2} \end{pmatrix}. \end{aligned} \tag{13.2}$$

The density matrices formed from these state-vectors are readily checked to be *momentum agent-states in disguise* as a superposition of position states, (Chapter 3, "Superposition and Context"):†

$$\begin{aligned} \mathsf{p}_1 \equiv |+\rangle\langle +| = \quad |\mathsf{p}, 1\rangle\langle \mathsf{p}, 1| &= \tfrac{1}{2}\begin{pmatrix} 1 & e^{ik(x_2-x_1)} \\ e^{-ik(x_2-x_1)} & 1 \end{pmatrix}, \\ \mathsf{p}_2 \equiv |-\rangle\langle -| = \quad |\mathsf{p}, 2\rangle\langle \mathsf{p}, 2| &= \tfrac{1}{2}\begin{pmatrix} 1 & -e^{ik(x_2-x_1)} \\ -e^{-ik(x_2-x_1)} & 1 \end{pmatrix}. \end{aligned} \tag{13.3}$$

These are the two possible states in the registers of the momentum agent. Following the Wheeler Principle, these agent-states become *physical* when they are observed. I turn now to their observation.

*The two states $|+\rangle, |-\rangle$, in the case of photons, correspond to horizontal and vertical polarization states. In the case of electrons, they correspond to "up" and "down" spin states. These are momentum states "in disguise."

†This superposition of agent-state-vectors and their associated density matrices(13.3) may be contrasted with an equal superposition of particle-beam density matrices (13.1) at each slit—an incorrect application of the superposition principle. Such a *faux* superposition of particle density matrices at each slit produces the fully random state and a position agent-actor (which does not satisfy the unit trace condition):

$$\frac{1}{2}(\mathsf{q}_1 + \mathsf{q}_2) = \mathbf{i}, \qquad \frac{1}{2}(\mathsf{q}_1 - \mathsf{q}_2) = \frac{1}{2}\mathsf{Q}.$$

a stark contrast to (13.3).

The Observer

Since the particle-beam is often displayed on a screen, I shall call the observer "the screen." The observing screen is a qubit agent O of a given species with a state of being I shall take as a position species with states,

$$O = \mathsf{q}_1, \qquad O = \mathsf{q}_2. \tag{13.4}$$

As described previously (Chapters 4, 5), the object of observation of canonical agents is properly the Hamiltonian of the system. Rather than illustrate a Hamiltonian that is a complex function of the observer O and the canonical agents, q, p, I divide the Double Slit system into two sets of agents: an "internal" set consisting the particle-beam agent integral to the mechanics of the beam and the slits, and an "external" agent that observes these internal agents. I summarize the key observations of the screen observer O as directly made on the particle-beam state, rather than the Hamiltonian function of that state. Consistent with this division, the screen agent is not entangled with the particle-beam agent.

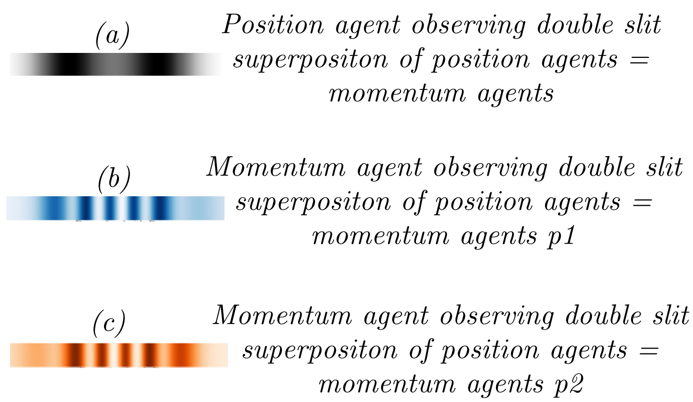

Figure 8-1. Observing Screen of The Double Slit. The agent-states at the exit of the double slit are momentum states which appear to position state observers as completely random (a). To momentum state observers, an interference pattern is observed which is either right (b) or left (c) polarized depending upon which of the two states in Eqs.(13.3) is being observed.

With both slits open, an interference pattern may or may not be observed: it depends on the observer. For the interference pattern to appear on the screen, the screen agent must be of the same species as the particle-beam agent. (Figure 8-1. Observing Screen of The Double Slit.) Given momentum states incident on the screen Eqs. (13.3), an observer $O = \mathsf{q}$ in a position context observes a random or so-called "clump distribution" (Fig. 8.1 (a).

$$(\mathsf{p}_1 \parallel \mathsf{q}) = \mathbf{i}, \qquad (\mathsf{p}_2, \parallel \mathsf{q}]) = \mathbf{i}.$$

An observer in a momentum context observes interference patterns corresponding to the patricular polarization state, p_1 or p_2 being observed (Fig. 8-1 (b), (c)):

$$(\mathsf{p}_1 \parallel \mathsf{p}) = \mathsf{p}_1, \qquad (\mathsf{p}_2, \parallel \mathsf{p}]) = \mathsf{p}_2$$

Double Slit with Detection

Let us continue the discussion of the Double Slit with inclusion of (1) Detectors that determine the slit through which the particle passes and (2) A focus on the observer with an "eraser" circuit in which the interference pattern can be made to appear and disappear by choice of the quantum species of the observer.

The Double Slit with detection introduces another degree of freedom, another particle—another qubit—into the picture. An indicator qubit embodies the detector. I now refer to the qubit associated to the particle traversing the slits as the *signal* and the qubit associated to the detector particle as the *indicator*. The indicator qubit identifies the path the signal particle followed. The indicator achieves its monitoring of the signal by being entangled with it.

I illustrate a system in which the particle beam is comprised of photons. (Other quanta, such as electrons, will behave similarly with respect to their observational behavior.) A typical source of an entangled pair of photons is a laser pump, a double-slit, and a parametric down converter, such as an $SPDC$ crystal. [28] This apparatus can be summarized abstractly as a source of entangled photons.

Indicator particle states, like those of the signal particle, correspond to the two slits, but indicate passage of the particle through the slit ("state of the slit") rather than the state of the passing particle ("state of the particle"). The indicator particle is a qubit with 2 states, which may be taken as position agents $|\mathsf{q}', 1\rangle$ and $|\mathsf{q}', 2\rangle$ from their origin in the $SPDC$. As these agents proceed through the experimental apparatus, they will be transformed to momentum agents. I will shortly describe how the indicator is implemented.

The case in which only one or the other slit is open, as discussed previously (see Double Slit Without Detection), is unchanged by the presence of detectors (a single slit open or closed is a *de facto* detector).

Particle-Beam Incident on Detector Array

Let the created pair have state $|\mathsf{q}\rangle|\mathsf{q}'\rangle$ where $|\mathsf{q}\rangle$ is the state of the signal qubit and $|\mathsf{q}'\rangle$ that of the indicator qubit. Both the A and B atoms are assumed to freely emit photons of both polarization states, q_+ and q_-, arbitrarily in position context. The state of the photons is therefore an equal superposition,

$$|\mathsf{q}\rangle = \frac{1}{\sqrt{2}}(|\mathsf{q},1\rangle + |\mathsf{q},2\rangle) \qquad |\mathsf{q}'\rangle = \frac{1}{\sqrt{2}}(|\mathsf{q}',1\rangle + |\mathsf{q},'2\rangle),$$

so the tensor product state-vector of the entangled pair is:

$$|\mathsf{q}\rangle|\mathsf{q}'\rangle = \frac{1}{2}(|\mathsf{q},1\rangle|\mathsf{q}',1\rangle + |\mathsf{q},1\rangle|\mathsf{q}',2\rangle + |\mathsf{q},2\rangle|\mathsf{q}',1\rangle + |\mathsf{q},2\rangle|\mathsf{q}',2\rangle.$$

The $SPDC$ created states $|\mathsf{q}',1\rangle$ and $|\mathsf{q}',2\rangle$ may be assumed to be non-overlapping with a vanishing inner product. The product states $|\mathsf{q},1\rangle|\mathsf{q}',1\rangle$ and $|\mathsf{q},2\rangle|\mathsf{q}',2\rangle$ are thereby orthogonal. This gives a tensor product state of the entangled pair in position context at the exit of the $SPDC$ without cross terms.

$$|\mathsf{q}\rangle|\mathsf{q}'\rangle = \frac{1}{\sqrt{2}}(|\mathsf{q},1\rangle|\mathsf{q}',1\rangle + |\mathsf{q},2\rangle|\mathsf{q}',2\rangle).$$

The corresponding density matrix is

$$\mathsf{q}\mathsf{q}' = |\mathsf{q}\rangle|\mathsf{q}'\rangle\langle\mathsf{q}|\langle\mathsf{q}'| = \frac{1}{2}[\mathsf{q}_1\mathsf{q}'_1 + \mathsf{q}_1\mathsf{q}'_2 + \mathsf{q}_2\mathsf{q}'_1 + \mathsf{q}_2\mathsf{q}'_2]. \qquad (13.5)$$

This is the state of the photons at the exit of the $SPDC$ crystal, that enter the detector array.

Preparation of Detectors

Detectors form a "photon titration" system that separates out the states of the entangled pair, (Fig. 8-2. Observing Screen With Detectors) The five detectors form an array through which photons in any of the entangled states (13.5) pass. The major titration is that of signal and indicator photons. Detector D_0 receives only signal photons. Detectors $D_1 - D_4$ receive only indicator photons.

Detectors are filters that block photons in states not keyed to a given detector. The state a detector indicates when it clicks corresponds to its preparation to filter out all but that given state. As photons in a particular state make their way through the array of detectors, each photon eventually encounters at least one detector that accepts it. That detector "clicks " identifying the agent-state that "clicked" it. If more than one detector clicks, the agent must have been in superposition of states which the detectors randomly decohere at each click. Individual detector records, once obtained, can be studied for correlations. Let us review them.

The incident states on detector D_0 are momentum states on the signal bit (a superposition of signal position states) and position states on the indicator bit (no superposition):

$$|\mathsf{p}_1\mathsf{q'}_1\rangle, \qquad |\mathsf{p}_1\mathsf{q'}_2\rangle$$

Before a particle passes through the slits, the indicator photon can be in an arbitrary resting state, $\mathsf{q'}_0$. After particle passage, the indicator photon enters either state $\mathsf{q'}_1$ if the particle passed through the first slit or $\mathsf{q'}_2$ if the particle passed through the second. The signal photon similarly passes from some initial state p into the Double Slit system where it takes either of its two (polarization) states $\mathsf{p}_1, \mathsf{p}_2$.

The evolution of the states at the exit of the Double Slit entering detectors D_0, D_1, and D_2 are then

$$\begin{aligned} \mathsf{qq}'_0 &\to \mathsf{p}_1\mathsf{q'}_1, \\ \mathsf{qq}'_0 &\to \mathsf{p}_1\mathsf{q'}_2, \end{aligned} \tag{13.6}$$

The entering states to detector D_3 and D_4 are

$$\begin{aligned} \mathsf{qq}'_0 &\to \mathsf{q}_1\mathsf{q'}_1, \\ \mathsf{qq}'_0 &\to \mathsf{q}_2\mathsf{q'}_2, \end{aligned} \tag{13.7}$$

(An agent-state without designation, such as q appearing in the the tensor product $\mathsf{qq'}_0$ may be in either of its states, q_1 or q_2.) A typical scenario from this set, such as $\mathsf{qq'}_0 \to \mathsf{q}_2\mathsf{q'}_1$, is to be read, "a signal photon in a position state q and entangled with an indicator photon in the resting state $\mathsf{q'}_0$ evolves through the double slit to a signal in the polarization state q_2 entangled with an indicator in the polarization state $\mathsf{q'}_1$."

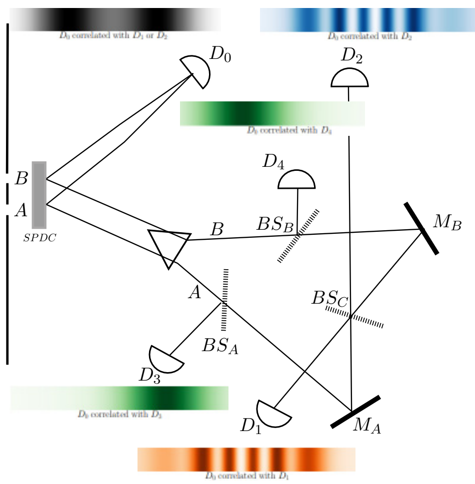

Figure 8-2. Observing Screens with Conventional Detectors $D_3 - D_4$ and Eraser Circuit with detectors $D_1 - D_2$. D_0 is a solitary detector receiving signal photons (in contrast to the pairs $D_1 - D_2$ and $D_3 - D_4$ which are capable of distinguishing individual polarization states). Since D_0 is triggered by both polarization states, its self-correlation $D_0 - D_0$ is a completely random distribution of "up" and "down" polarization states. Detectors $D_3 - D_4$, receive the indicator in the fashion of the conventional double slit on paths $A - D_3$ and $B - D_4$, each triggered by one or the other of the indicator's polarization states. These detectors, intercepting different paths, obtain "which path" information and a "clump" pattern without interefer-ence. The Eraser Circuit consists of Mirrors M_A, M_B directing photons to detectors $D_1 - D_2$ intersecting at beam splitter BS_C. Detector pair $D_1 - D_2$ receives indicator photons on paths $A - C - D_1$ and $B - C - D_2$ which are mirrored into an equal superposition in the eraser circuit. The superposition *puts them into a momentum state* that triggers D_1 or D_2 depending upon the indicator's polarization. This change of the indicator's context from position to momentum is the secret to the seemingly magical disappearance and reappearance of the interference pattern in the "eraser." The superposition induced in the eraser circuit has removed the possibility of obtaining "which path" information. Note that the only thing that is different (no change in the source S) is the observer: in the conventional case the observer $D_3 - D_4$, in the "eraser" case $D_1 - D_2$. [Interference patterns from Fankhauser [30]].

DetectorD_0 is triggered on either of the polarization states of a signal photon. D_0 is a solitary detector (in contrast to the pairs $D_1 - D_2$ and $D_3 - D_4$ which are capable of distinguishing individual polarization states). Since D_0 is triggered by both polarization states, its self-correlation $D_0 - D_0$ is a completely random distribution of "up" and "down" polarization states.

Two remaining detector pairs receive the indicator photon. One pair, D_3 or D_4, receives the indicator in the fashion of the conventional double slit on paths $A-D_3$ and $B-D_4$, each triggered by one or the other of the indicator's polarization states. The other pair receives the indicator on paths $A-C-D_1$ and $B-C-D_2$ which are mirrored into a superposition in the eraser circuit (Fig. 8.2). The superposition *puts them into a momentum state* that triggers D_1 or D_2 depending upon the indicator's polarization. This change of the indicator's context from position to momentum is the secret to the seemingly magical disappearance and reappearance of the interference pattern in the "eraser." The interference pattern can be made to appear and disappear at will by choice of observer of the eraser detectors $D_1 - D_2$

13.0.1 Input States to Detectors

A typical set-up consists of five detectors (one solitary only receiving signal photons, the others in pairs receiving indicator photons). They form a "photon titration" system that separates out the states of the entangled pair. (Fig. 8-1). Correlations of individual detector records reveal this information.

D_0 is a solitary detector (in contrast to the detector pair D_3-D_4 which are capable of distinguishing individual polarization states). Detector D_0, triggered by either of the polarization states of a signal photon, is incapable of distinguishing them. Its triggering states are superpositions of the output states of the $SPDC$ which are position agents. As described in *Superposition and Complements*, the triggering agent for D_0 is thereby a *momentum agent*.

Detectors $D_3 - D_4$, accept the indicator in the fashion of the conventional double slit on paths $A-D_3$ and $B-D_4$, each triggered by one or the other of the indicator's polarization states. Detectors D_3-D_4 are prepared to "click" on one of the position input states

$$\mathsf{q}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \mathsf{q}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{13.8}$$

$D_1 - D_2$ is the detector system implementing the quantum eraser. Detectors D_1, D_2 are prepared to "click" on a momentum input state which can be one of two possible polarizations,

$$\mathsf{p}_1 = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \qquad \mathsf{p}_2 = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{13.9}$$

Equal superposition of the two paths in a position context immediately implies this is a momentum context. Accordingly, detectors $D_1 - D_2$ can gather no information indicating the slit from which the signal photon issued if they too are in a position context *given that the indicator originated in the same position context as the signal.*

The detector response to the incident states (4.16) may be summarized as

$$D_0 \text{ accepts } \mathsf{p}_1, \mathsf{p}_2$$

$$D_1 \text{ accepts } \mathsf{p'}_1$$

$$D_2 \text{ accepts } \mathsf{p'}_2$$

$$D_3 \text{ accepts } \mathsf{q'}_1$$

$$D_4 \text{ accepts } \mathsf{q'}_2$$

These detector states may be observed by an observer O.

Observation of Signal and Detector

Since the signal is often physically displayed on a screen and the indicator measured by a coincidence counter, I shall call the signal observer "the screen" and the indicator observer "the counter." Both are agents of a given species. Both have states of being. The two form another canonical quartet of tensor atoms.

$$O: \qquad \mathsf{q}\mathbf{i}, \quad \mathbf{i}\mathsf{q}', \quad \mathsf{p}\mathbf{i}, \quad \mathbf{i}\mathsf{p}', \tag{13.10}$$

where the screen holds the first bit and the counter holds the second. Tensor atoms for which tensor products may be observed without disturbing both bits have the random state **i** as a potential state of either the screen or the counter, as in

$$O = \mathsf{q}_1\mathbf{i}, \qquad O = \mathbf{i}\mathsf{p}_2.$$

As described previously, the object of observation of the canonical agents is properly the Hamiltonian of the system. Rather than illustrate a Hamiltonian that is a complex function of the observer O and the canonical agents, I divide the Double Slit system into two sets of agents: an "internal" set consisting of signal and indicator qubits integral to the mechanics of the beam and the slits, and an "external" set of agents O consisting of screen and counter qubits that observe the internal agents. I summarize the key observations of the screen-counter observer O as directly made on the signal-indicator state, rather than the Hamiltonian function of that state. Consistent with this division, the screen-counter tensor product is thereby dis-entangled from the signal-indicator tensor product.

With both slits open, an interference pattern may or may not develop: it depends on the observer. For the interference pattern to appear, the screen agent must be of the same species as the particle agent.

Let us review the various observations of the states appearing in (13.5) where, as before, the signal holds the left bit position, and the indicator the right bit. The incident states on detectors D_0, D_1, and D_2 are momentum states of the signal bit and position states of the indicator bit:

$$|\mathsf{p}_1\mathsf{q'}_1\rangle, \qquad |\mathsf{p}_1\mathsf{q'}_1\rangle$$

Before a particle passes through the slits, the indicator photon can be in an arbitrary resting state, $\mathsf{q'}_0$. After particle passage, the indicator photon enters either state $\mathsf{q'}_1$ if the particle passed through the first slit or $\mathsf{q'}_2$ if the particle passed through the second. The signal photon similarly passes from some initial state q_0 into the Double Slit system where it takes either of its two (polarization) states $\mathsf{p}_1, \mathsf{p}_2$. (Equal superpositions of entering position states become momentum states.)

The evolution of the signal is registered only by detector D_0 and as a superposition of position states which will be received as momentum states of the signal bit.

$$\begin{aligned} \mathsf{q}_0\mathsf{q'} &\to \mathsf{p}_1\mathsf{q'}, \\ \mathsf{q}_0\mathsf{q'} &\to \mathsf{p}_2\mathsf{q'}, \end{aligned} \tag{13.11}$$

The evolution of the of the indicator from the source at the exit of the Double Slit entering detectors D_1, and D_2 are also momentum states because of the superposition induced by the eraser (notice the momentum states are now on the indicator bit),

$$\begin{aligned} \mathsf{q}\mathsf{p}'_0 &\to \mathsf{q}\mathsf{p'}_1, \\ \mathsf{q}\mathsf{p}'_0 &\to \mathsf{q}\mathsf{p'}_2, \end{aligned} \tag{13.12}$$

The entering states of the indicator photon to detectors D_3 and D_4 maintain the same position context in which they were created,

$$\begin{aligned} \mathsf{q}\mathsf{q}'_0 &\to \mathsf{q}\mathsf{q'}_1, \\ \mathsf{q}\mathsf{q}'_0 &\to \mathsf{q}\mathsf{q'}_2, \end{aligned} \tag{13.13}$$

(An agent-state without designation, such as q appearing in the the tensor product $\mathsf{q}\mathsf{q'}_0$ may be in either of its states, q_1 or q_2.) A typical scenario from this set, such as $\mathsf{q}\mathsf{q'}_0 \to \mathsf{q}_2\mathsf{q'}_1$, is to be read, "a signal photon in the state q entangled with an indicator photon in the state q_0 evolves through the double slit to a signal in the state q and an indicator in the polarization state $\mathsf{q'}_1$."

Observation with Detection

I summarize the key observations of the screen-counter observer O as directly made on the signal-indicator state. For all cases in which the indictor photon *acquires no information*, the observing screen scenarios with detection will be the same as those without, Eqs. (13.14) - (13.15). If the signal holds the left bit position, and the indicator the right bit, as in the previous illustration, these will be observation events *by observers that are left tensor atoms* $O = \mathsf{p}\mathbf{i}$,

$$\begin{aligned} (\mathsf{p}_1\mathsf{q'}_1 \parallel \mathsf{p}\mathbf{i}) &= \mathsf{p}_1\mathbf{i}, \\ (\mathsf{p}_1\mathsf{q'}_2 \parallel \mathsf{p}\mathbf{i}) &= \mathsf{p}_1\mathbf{i}, \\ (\mathsf{p}_2\mathsf{q'}_1 \parallel \mathsf{p}\mathbf{i}) &= \mathsf{p}_2\mathbf{i}, \\ (\mathsf{p}_2\mathsf{q'}_2 \parallel \mathsf{p}\mathbf{i}) &= \mathsf{p}_2\mathbf{i}, \end{aligned} \tag{13.14}$$

The signal photon and its interference pattern are observed, but no which-path information is obtained from the indicator photon.

To find out information about the indicator photon, the observer must be a right tensor atom $O = \mathbf{i}\mathsf{q}$. In the following observation scenarios the observing right tensor atom $\mathbf{i}\mathsf{q}$ may be in either polarization state $\mathsf{q'}_1$ or $\mathsf{q'}_2$. The observation event will evolve it to the state of the indicator photon:

$$\begin{aligned} (\mathsf{p}_1\mathsf{q'}_1 \parallel \mathbf{i}\mathsf{q}) &= \mathbf{i}\mathsf{q'}_1, \\ (\mathsf{p}_1\mathsf{q'}_2 \parallel \mathbf{i}\mathsf{q}) &= \mathbf{i}\mathsf{q'}_2, \\ (\mathsf{p}_2\mathsf{q'}_1 \parallel \mathbf{i}\mathsf{q}) &= \mathbf{i}\mathsf{q'}_1, \\ (\mathsf{p}_2\mathsf{q'}_2 \parallel \mathbf{i}\mathsf{q}) &= \mathbf{i}\mathsf{q'}_2. \end{aligned} \tag{13.15}$$

Which-path information has been acquired by the atomic observing agent $\mathbf{i}\mathsf{q}$: the state of the indicator photon has been found out. But no information has been carried away from the signal photon by this observer. The interference pattern of signal photons is intact. These results may be formulated in this important principle

Principle (Which-Path vs. Interference)**.** *The observer of a double slit experiment can obtain which-path information the photon follows from the indicator bit in which case the observation screen displays a random pattern from the signal bit. Conversely, the interference pattern may be observed from the signal photon, but which-path information obtained from the indicator is completely random. Such is the reality created by the entanglement of this photon pair.*

As described in Chapter 8, the *Principle of Selective Observation,* agents in complementary contexts observing one another fail to do so; they carry away no definite information, only completely random information:

$$(\mathsf{qp} \parallel \mathbf{i}\mathsf{p}) = \mathbf{i}\mathsf{p}, \qquad (\mathsf{p} \parallel \mathsf{q}) = \mathbf{i},$$

It is in this sense that the information is "erased" but it is more appropriate to say the information holding the interference pattern *cannot be observed* in the (complementary) state of the eraser. The information is still held by the entangled pair—and can be revealed by selective observation by an agent of the same species as follows.

Principle (Indicator Photon State Concealed as Signal State is Revealed)**.** *Since the indicator state is entangled with the signal state, a measurement of the indicator reveals the state of the signal. Although the indicator photon state is destroyed in measuring it, the entanglement correlation gives a measurement of the signal photon without disturbing it. Such is the observation magic of entangled tensor states. This is the critical aspect of this experiment which preserves interference patterns even in the face of observation/measurement.*‡

To Conceal or Reveal? The Quantum Eraser

The "quantum eraser" [28] is misnamed. It would be more appropriately called the *quantum concealer* as it does not erase information; it conceals it from certain observers (those of a different species from the signal) and reveals it to others (those of the same species). (See "Species and Varieties of Agents" in Chapter 3). This is not the end of the story. I showed that for agents in tensor product states, information can be revealed for one of the agents of the product and concealed for the other (Chapter 8, Selective Observation of Tensor States). This phenomenon is captured in the Double Slit system in the principle "Which-Path vs. Interference."

Concealing and revealing simply turn on whether information has been extracted from the indicator or not. Specifically, for the interference pattern to appear, the indicator bit of the must be of a complementary species from the observer in detectors $D_1 - D_2$ so as not to carry away any information.

The quantum eraser configuration (Figure Eraser Configuration) is the Double Slit With Detection arrangement discussed previously (Figure Double Slit) modified to an Eraser Configuration. In the Eraser Configuration, beamsplitters BS_A, BS_B are introduced that open up new paths to an added pair of detectors $D_1 - D_2$. Half the events at the beamsplitters BS_A and BS_B proceed as in the case of Double Slit With Detection on paths $A - D_3$ and $B - D_4$.

For the other half of the events at BS_A and BS_B, Photon 2 is reflected to beamsplitter BS_C where it follows either path $A-C-D_1$ to detector D_1 or path $B-C-D_2$ to D_2. These paths converge on and interfere at the quantum eraser at BS_C shown in Fig. 10-2. It is important to keep in mind that Photon 2, as it traverses a more

‡This behavior is similar to the destruction of the state of an original quantum when it is quantum-teleported to a new location. See [?] and the references cited therein.

complex set of paths and encounters with beamsplitters and mirrors, remains an entangled partner to Photon 1 which follows a direct path to detector D_0. The entanglement is not destroyed until Photon 1 clicks detector D_0 or Photon 2 clicks detector set $D_3 - D_4$ or $D_1 - D_2$.

Examine in more detail the operation of the eraser (Figure 8-2). The key to the eraser's action is its ability to put photons into a superposition of the two paths, $A - C - D_1$ and $A - C - D_2$. Beamsplitter BS_C and its associated mirrors M_A, M_B accomplish this by both reflecting and transmitting photons.

To see how the eraser superposition is brought about, consider a photon coming off mirror M_A and entering beamsplitter BS_C. BS_C transmits that photon into Detector D_2 or reflects it into detector D_1 with equal probability. In the same manner, a photon coming off Mirror M_B and entering BS_C is either transmitted to detector D_1 or reflected to D_2 with equal probability. A photon in the mirrored eraser circuit $M_A - BS_C - M_B$ is thereby in an equal superposition of the two paths to detectors D_1 and D_2.[§] By the Principle of Superposition and Complementarity (Chapter 3), these photons, originating as position agents, have evolved in the eraser to become momentum agents. This is in contrast to photons on paths $S - D_0$, $A - D_3$, and $A - D_4$ which remain position agents where S is the source of photons at the exit of the $SPDC$.

The Quantum Eraser is a Quantum Concealer which can be made to reveal or disappear an interference pattern by choice of observer species relative to signal species. Choosing an observer species that is the same as that of the signal reveals. Choosing a complementary species conceals.

Delayed Choice

Time was not relevant to the previous discussion. It was implicitly assumed that the observer's observations of the detectors were simultaneous. I will now remove that assumption. The times at which detectors $D_1 - D_4$ fire will be assumed to be simultaneous; the time at which detector D_0 clicks and can be made much earlier by implementing a delay line over which the indicator must travel to reach $D_1 - D_4$. In the experiments reported by Kim, et. al this distance is 2.5 meters and the delay can be as much as 8 ns between the time the the signal photon strikes D_0 and the time the indicator photon strikes D_1 or D_2.

The signal photon has a simple path from the source to detector D_0 prepared in a position context, the same context as the photon incident on the $SPDC$ crystal. The "delay" in this experiment is implemented by a long optical path for the indicator

[§] This is a good illustration of how the technology of glass surfaces and coatings that change the index of refraction can be used to put quanta into states of superposition.

photon to transit from its source to the detector set $D_1 - D_4$. Signal and indicator photons leave the $SPDC$ crystal simultaneously. The signal photon on the short path to detector D_0 is observed first in either the state $|\mathsf{p}, 1\rangle$ (or $|\mathsf{p}, 2\rangle$). The indicator photon, undisturbed and unobserved, continues in the superposition (13.3) in which it was placed (by passage through the slits) until it reaches the detector set $D_1 - D_4$ after a delay.

Standard quantum theory posits a global wave function. In the delayed choice configuration, the collapse of a global wave function triggered by the signal photon at detector D_0 implies collapse of its entangled mate in transit to $D_1 - D_2$ *before its arrival*. This has led to incoherent claims that "information goes backward in time." The Wheeler Principle asserts, instead, the state of the indicator photon is not established until it is observed. The scenario respecting this principle is as follows.

Following its state creation in the double slit and the $SPDC$ crystal, the indicator photon remains in the superposition (13.3) until it is observed whereas the signal photon has been observed in either the state $|\mathsf{p}, 1\rangle$ (or $|\mathsf{p}, 2\rangle$. When the indicator photon finally reaches $D_1 - D_2$, it collapses to a definite momentum state of that detector, $|\mathsf{p}, 1\rangle$ or $|\mathsf{p}, 2\rangle$. Either of these states is consistent with the entangled signal photon in the superposition $|\mathsf{p}_{1,2}\rangle = \frac{1}{\sqrt{2}}(|\mathsf{q}, 1\rangle \pm |\mathsf{q}, 2\rangle)$ (Eq. (13.3), which, on observation, collapses to a mixed state of equal proportions of $|\mathsf{q}, 1\rangle$ and $|\mathsf{q}, 2\rangle$: the fully random state $\mathbf{i}$.

Wheeler Comment

"The past has no existence except as it is recorded in the present [Wheeler (1978)].

Summary

Tensor states and the "software" tensor chemistry of automata introduced here naturally describe the flow of information among entangled agents. Tensor agents clarify the observation of entangled agents epitomized in the quantum eraser in which interference patterns can be made to appear and disappear depending upon the observer. The phenomena are still quantum effects, but extravagant claims, such as those made for "signals going backwards in time" in the case of the quantum eraser, are explainable as simple consequences of the Wheeler and Peres observation principles—once one has a perspective of quantum agents and the framework of quantum automata.

Chapter 14

Randomness

Random events observed by human agents in the flow of life are familiar. If you found yourself in a strange city and knew nothing about the schedules of trains and buses, their times of passing might seem random and unpredictable. They become predictable if you gain access to their timetable. You now hold information you did not have when they appeared randomly. You could also observe their passage day by day and record their times thereby building up an experimentally observed timetable. In both cases, randomness is dispelled by new information: the timetable.

Principle (Randomness)**.** *Random events and the states in which they manifest do not exist in isolation from observation. The phenomenon of randomness requires both events and observers: states and observers of those states. According to the Peres observation principle, different observers may record different observations*

Information is always with respect to an agent that observes, "possesses" or "holds" information that defines, describes, or is associated with the event. Randomness experienced by an agent and the presence or lack of information held by that agent go hand in hand. In the case of quantum automata, agent registers hold the agent's information: its state of being ("Quantum Automata," Chapter 3). Agents gain information about other agents by observation of the community of agents encoded in the Hamiltonian superagent. That information is expressed in its impact on the agent's motion: its new state in time. That new state is open to random information because of the essential Heisenberg uncertainty of the observed Hamiltonian.

Our experience of dispelling randomness with new information leads to the speculation that randomness is a matter of perception, of definite information available to some observers but not others. According to this belief, somewhere, somehow, possibly in "a realm of the gods beyond," a divine agent holds all events *and* the information that defines them. What appeared to us (observers) as random was actually a lack of information on our (observer's) part. According to this belief, the dynamics of the universe are, however incomprehensibly complex, deterministic to an omniscient observer.

The discovery of the quantum denies this speculation. There exist physical events the randomness of which cannot be dispelled by the most determined efforts to expose "a deterministic cause". Beginning with Heisenberg's discovery of the uncertainty principle, unpredictable behavior has been an inevitable presence in quantum physics ("Heisenberg Uncertainty Principle," Chapter 7). Unpredictable behavior manifests itself as *random* behavior. The qubit automaton, the most elementary quantum system with a canonical pair of quantum agents, bears the random deep within its structure. Only one of its two bits of information is accessible to *definite* observation by *any* agent. The other bit reveals itself randomly. As pointed out in Chapter 6, the universe of an isolated qubit automaton balances the maximum amount of order with the maximum amount of randomness in a single degree of freedom.

Algorithmic Randomness

Natural computation [8] in concert with observation plays a central role in Autonomous Quantum Theory. The two meet in the Principle "Observation generates the motion of observers," (Observation as Motion, Chapter 6). There is another form of randomness that emerges from the computation of numbers, namely, *Algorithmic Randomness* described by Algorithmic Information Theory (AIT).

Algorithmic randomness is a property of the Turing machine and related mathematical abstractions that describe computation, such as the Lambda and Combinator Calculi. Randomness emerges from the Turing machine in a failure to halt on some inputs. Such inputs are sad to be *uncomputable*. It cannot be proved that a possibly uncomputable input is either computable or uncomputable without running the Turing machine until it halts. Such a procedure must be open to an infinite number of computing cycles.

"Definite state" designates a a definitely observed state in quantum physics. A "definite object" in AIT is a "computable number."* The corresponding "random object" is an *uncomputable number*. The reception of a definite observation by a quantum agent in quantum physics corresponds to a Turing machine that halts with a definite result in AIT.

So we have a correspondence between autonomous quantum agents and their states, definite and random, and abstract Turing machines and their states, definite (computable) and random (non-computable). One set exists in the physics of nature, the other exists in human minds and the productions of human minds. What is the relationship between them? A lively literature exists on this subject [38], [40]. Sur-

*Information is fungible. An information string can always be encoded as a number.

prisingly, none of this work, interesting as it is, focuses on the role of the brain and mind, individually and socially, as essential to AIT. Human brains and minds (or their equivalent) are essential to support Algorithmic Randomness and AIT. Human brains and minds are not essential to Quantum Randomness. Quantum randomness is indigenous to the universe in matter;

Turing Machine Embodied: Brain and Mind in Natural Computation

The Aristotelian principle of matter and form with *matter* understood as atoms, molecules and other elementary particles and *form* as the information that specifies the state of these particles is fundamental to natural computation.†

Principle (Duality of Brain and Mind)**.** *The Turing machine exists in the physiological and neurological "hardware" of the brain and the software of the mind encoded in it. Individual minds themselves extend into a wider community of minds and the productions of the community of minds that constitute a collective consciousness.*

For human beings, the natural computation of the mind is principally located in the brain and nervous system. The neural structures of the brain encode information beyond the fundamental encoding of the dynamics of the elementary particles that compose its matter. [41, 42, 43] ‡ (See "A Coding Theorem," Chapter 6.) Neurons and their associated neural neural structures and their dynamics encode these mental states as directed by their governing Hamiltonians. Individual brains and minds form the collective mind in the interacting society of individuals and their productions: a culture or "collective consciousness."

Consider an individual mind in the social milieu of a community of minds and culture. The concept of the Turing machine, however stimulated by ideas circulating

†To illustrate: A lipid molecule consists of approximately 100 C, N, O, H, and P atoms. That is the lipid matter in mass-energy. These atoms are formless until bonded together in some molecular structure. They could be other fatty hydrocarbons, or several other kinds of smaller molecules, as they are cooled, qua matter. Chemically bonded into a lipid molecule, the position states of these atoms fix the quantum state of the position coordinates—a complex state specification indeed. Next, consider the momentum coordinates, bearing, first of all, the translational and rotational states of the center of mass, then the vibrational, rotational, and bending modes of the many internal degrees of freedom, an equally complex state specification of the molecule's motion. Nonetheless, all together they proclaim "a lipid molecule in a certain position in space and time in the universe with a certain momentum." The chemically bonded position states write "lipid" in a language of atomic letters and molecular sub-unit words. The momentum states tell how that lipid is moving. The lipid as whole, both in structure and motion, can be regarded as an *informational* expression written in a chemical language.

‡Important brain regions implicated in its ability to encode mathematical concepts and operations include the fusiform gyrus and the intraparietal sulcus. Hippocampal frontal circuits execute the formation of associative memories and the binding of new and old information in executing mathematical thought.

at the time of "what it means to compute,"[§] first emerged in the mind of Alan Turing. Turing's concept of a sequence of discrete machine states and a tape with cells that can be read, written to, re-written to, and moved, according to the machine state and an evolving code on the tape was enormously prescient. The motion,

$$\textit{observe-compute-move-repeat-}\cdots,$$

mimics the essential operation of the underlying paradigm of natural computation present in the qubit automaton (Chapter 3, Chapter 6)

Turing's mind, of course, was part of a wider intellectual community socially extended in brains and minds that included Emil Post, Stephen Kleene, Alonzo Church, and others. Many minds nourished Turing's thinking: his definition of a computable function as a Turing machine program that effectively reaches completion in a finite number of steps proved to be an effective insight into the ways of thought and, more generally, the ways of nature. Nature, in her dynamics, taught Alan Turing and a community of mathematicians the machine that now bears his name.

A major difference between AIT computation and natural computation is the TM, as Turing conceived it, halts and completes a computation if the function is computable, else it never halts. A mind can do this for a definite (halting) state (generally with the aid of scratch paper) and conceive of a non-halting state even if it is impossible to complete all the steps. Dynamic computation in nature in quantum automata is continuous with random states produced and entered into the stream of agent observations as time advances (Chapter 6).

Principle (Turing Machine in Brain and Mind)**.** *The Turing machine running as a mental structure in the mind is physically instantiated in the biology of the brain. Computations executed in brains and minds obey Landauer's principle: the information is encoded in matter—neural cells and chemical networks. Socially, individual brains extend into culture and networks of minds that make a community of mathematicians, and beyond, a wider culture. It is accurate to say a human being executing the steps of a Turing Machine in his or her mind (likely with the help of pencil and paper to off-load intermediate results) has a Turing machine running in the mind (see the fine illustration of Roger Penrose [27].)*

We see uncomputable functions and the randomness they imply emerge from the abstraction of the Turing machine. But the Turing machine exists in the hardware underlying minds: the brain and its social extensions into the community of brains and minds, fleshy and physical. A sharp distinction between algorithmic randomness as an abstraction and quantum randomness as physical cannot be made. Rather, I believe the side-by-side existence of quantum randomness and algorithmic

[§]Beginning with the axioms of Arithmetic of Giuseppe Peano and the notion of recursive operations (applying a function to itself) randomness an example of what mathematician V. I. Arnol'd called "the mysterious unity of all things."[45]. The distinction that can be made is that the former is a fundamental property of the universe while the latter is a late development of evolutionary life on planet earth.

Algorithmic randomness is sweeping in its generality. The random turns on the capacity for self-reference and is independent of the semantic content of the sets. Most importantly, algorithmic randomness takes no account of physical laws, qua law, that bear on the content of these sets.

Physical World and the Universal Turing Machine

A Universal Turing Machine (UTM) simulates any Turing machine by taking as input the code describing the given machine joined to the given machine's input. The Turing machine's capacity to process representations of the most general kind includes representations of itself and its kin. The Quantum No-Cloning Theorem shows this is not always possible in a quantum universe.

Can the Universal Turing Machine be a model for building up ordered nature from random bits [8]? The model is "monkeys typing on keyboards," a metaphor for completely random input.¶ If monkeys type randomly on disconnected keyboards, nothing is produced. But if the keyboard is connected to a UTM, the monkeys' random typing is Turing machine input. Eventually they produce every possible bit string as input. The computer, as a Universal Turing Machine, acts on this input. Most of these random strings cause the UTM to hang or go into an infinite loop. But some of the strings necessarily generate definite and meaningful output because among all possible strings are those strings that do indeed produce definite and meaningful output, such as the sum of a pair of numbers, a sequence of the digits of π, a DNA sequence of Hemoglobin, the text of Shakespeare's *Hamlet*, and so on.

Quite aside from the constraints of the No -Cloning Theorem (Appendix E) on such gratuitous computation, a purely mathematical model of algorithmic randomness is not compelling as a deep principle for the natural world: it does not account for the pervasive presence of physical law, in particular, for its encoding in a Hamiltonian dictated by the configuration of particles and force laws.

The structure of Hamiltonian mechanics is an effective and powerful ordering principle that quantum automata possess in the explicit presence of the elementary force laws and the Hamiltonians that encode them. The Turing machine model only includes these things by its universal property of "including every possible information string." Equations of motion have the same status as a recipe for chicken soup, or

¶ *All* possible strings are eventually included in the monkeys' input. If an input string contains n bits, the number of possible strings is 2^n—an exponentially large universe of strings.

even a string of gibberish, as "just another string." To include every string in general is to include no string in particular, a weak connection to nature compared to the *direct physical connection* in force-law and Hamiltonian.

Order from Quantum Randomness

How do naturally created random bits in quantum automata observational dynamics get reprocessed as new definite bits? This is a challenging problem. The emergence of life on earth from pre-biotic chemical precursors is a kindred—and similarly challenging—problem. Here, I offer a few guiding principles.

To see the emergence of new states and agents beyond the qubit, do not regard the qubit automaton as the universe of a single degree of freedom. Rather, take the universe itself as the fundamental state space for which one degree of freedom may, in certain circumstances, be singled out and isolated (as in the Stern-Gerlach spin experiments). It certainly is possible in the environment of a physics lab on the earth to do this with quite effective isolation. From this perspective, nature as a whole is given and the qubit automaton (a single freedom) singled out by isolation of one kind or another.

As isolating structures are lifted, more freedoms come into play. The isolated qubit automaton can then be seen in a wider community of tensor product automata. The picture becomes one of a universe of possibility in its many states, some of which are filled, some empty. States become excited and de-excited as time evolves. Condensed matter opens the way to unimaginable creative possibilities of rich structures of tensor automata.

The Quantum Origin of New Information and an Open Future

In a world of bits, there is nothing new under the sun. Bits are passed through to new states under the guidance of the Law of Motion determined by the values of present bits—ultimately determined by some primal initial condition. The common phrase for this process is "the deterministic universe."

The quantum world of qubit automata is richer. Present qubits do not determine future qubits: an unpredictable stream of new information enters the computation of motion at each step as ordained by the Heisenberg Principle.

In the quantum world of qubits, a possibility exists for spontaneous new

> information to be born in every quantum interaction; present bits do not rule the future. Unpredictable quantum motion, once it is observed and entrained in the time-evolution operator, becomes definite in the time-evolved information string of a qubit as *new information*.
>
> The random mutation in the quantum motion of DNA copying is an engine of evolution, the spark of life. Randomly changed DNA qubits are new creations. Random bits are raw material, free bits for the use of ordered bits evolved in more complex molecular structures at a higher emergent level. Natural selection uses the random bits of DNA copying to refine new stable life forms made possible by the opening up of a random transition that was not there before.

In a world of qubits, randomness is an essential feature. Randomness for human beings attempting to design or navigate life to a goal is often regarded as an annoyance, a defect, deleterious to design. We popularly think of random as 'bad' (though we welcome random good fortune when it surprises us). Quantum randomness is all these things but more deeply a marvel of nature without which the universe would be less vibrant than a cold and lifeless crystal.

Summary

Two fundamental principles underlie the theory of quantum automata. According to Wheeler's Principle, quantum agents are not isolated entities. They exist in community with other physical agents and acquire physical existence in their interaction with them. What we call "observation" is better understood as "interaction" embedded in the dynamic of motion itself. The Peres Observation Principle complements Wheeler's principle with the proposal that different agents do not necessarily register the same observation.

The existence of multiple observation records is intrinsic to quantum nature as Peres pointed out. In concert with Heisenberg uncertainty the two principles frees quantum evolution from the lock-step deterministic grip of unitary evolution of *observed* canonical agents. At the same time unitary invariance is maintained as a foundation principle: the evolution of the Hamiltonian superagent is unitary, preserving order in nature. Together these principles lie at the foundation of quantum dynamism: a stream of unpredictable information constantly enters the stream of definite information, a source of novelty, creativity, vitality.

The statistical stasis underlain by unpredictable dynamics implied by The Fundamental Theorem (Chapter 7) drops a curtain over the inner working of nature. Quantum dynamics are not fully accessible to a deterministic theory of motion or even to the various approximations, such as those that add ad-hoc statistical terms to the Schrödinger equation. This might seem a limitation to the description of the physical world.

What is changed is the recognition that agents that operate according to those laws have incomplete information of one another's states. Incomplete information is in the form of random bits in agent observations of the Hamiltonian. Agent actions are therefore carried out on the basis of some random information. This random information is the raw material for new creation: a wild universe within the constraint of physical laws open to the random. The result is a portrait of the natural world that brings quantum physics into greater accord with human (and sentient creature) experience of the natural world.

Chapter 15

Quantum Automata and the Fullness of Nature

This book is a beginning, not an end, nor even a midpoint. I have turned the page to a way of thinking about and exhibiting the dynamism of nature. Modern Quantum Mechanics is central to this understanding, but in a capacity for agency and automaticity that open the way to creativity in nature. A little over a century ago, it was unknown as a physical theory.

Autonomous Quanta conveys the theory of quantum automata with an irreducible element of quantum motion, the qubit automaton. There is no more elementary dimension of motion than a single degree of freedom. There is no more elementary dynamic and autonomous structure acting in that degree of freedom that observes and moves based on its observations than the qubit automaton.

The qubit automaton possesses the spark of autonomy—it observes and moves—that makes life possible. In itself the qubit automaton is far from a living organism in the same way a Carbon atom has the generous atomic valence structure that makes life possible, yet a solitary Carbon atom is very far from a living organism. The qubit automaton shares some of the singular features of atoms as Richard Feynman observed,

> If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis that all things are made of atoms.
>
> *R. P. Feynman*

The hypothesis of elementary agents in nature—qubit automata—is a similar hypothesis containing the most information in the fewest words that captures the dynamism of nature. A qubit automaton is the most elementary unit of motion:

"all things are informationally composed of qubit automata."

Qubit automata are the informational organizing agents of atoms, molecules, and other elementary particles ("what to be," "where to go"), as much a part of the particles as their mass, spin, and charge.

In quantum physics, random phenomena are inescapable, as Heisenberg first discovered and proved and Albrecht and Philips [1] elaborated. For quantum particles at the finest scale of a few degrees of freedom, the systems are too simple to support a complex computational reasoning system. At the level of animals with well-developed nervous systems, rudimentary reasoning necessary to their survival, such as a capacity to estimate the likelihood of being eaten while eating, exists. We don't know very well what is possible between these two extremes.

A solitary qubit automaton with a memory capacity of 2 bits surely cannot reason in the way we understand, for example, the operations of first-order logic. Its behavior is a combination of definite motion entwined with random motion which makes it an indeterministic agent—though it is important to bear in mind there is a good bit of determinism as well in the laws of motion and the Hamiltonian code. It receives the signals of the states of similar elementary automata and acts on them according to the directives of the Hamiltonian code. The Hamiltonian code, supplied by nature, is fundamental to Dynamism. It's the "DNA code" of mechanics and a forerunner of biological codes [26] emergent from it and an evolving environment.

As degrees of freedom become more numerous and tensor atoms and molecules aggregate into more complex structures, more extensive behavior can be expected to emerge. In abundant collections of matter of the appropriate chemical species, agents form and break chemical bonds building up more complex structures that ultimately become the world around us. The action would be utterly deterministic save for the fact that the observation of the Hamiltonian code is imperfect and partly suffused with randomness that foreshadows the way DNA code is partly suffused with randomness. The motion is therefore executed with some random elements. From a perspective of idealized mechanical determinism, this is a big mistake. But from an evolutionary perspective, for which randomness is essential, this is mechanical magic.

Turing's definition of "intelligence" turns on the capacity of agents to make mistakes. Quantum agents have the "capacity to make mistakes." One might say quantum randomness and its random mistakes are a higher form of perfection than deterministic, mechanical construction. Randomness is an essential component of evolution at the deepest level: "informational" fuel, the raw material of an engine of novelty and creativity. With Turing's definition as a reference, one can say a qubit automaton is intelligent—where the word ‘intelligent” is understood in its most primitive sense: the capacity to not only observe, but to act on one’s observations.

BITS IN THE PRESENT UNIVERSE
(LARGE STRUCTURES, MOLECULES, ATOMS, AND OTHER ELEMENTARY PARTICLES)

known universe: 10^{90} bits
all planets: 10^{75} bits
planet earth: 10^{50} bits
human being: 10^{28} bits
bacterial cell: 10^{14} bits
supramolecules: 10^{12} bits
human DNA: 10^{9} bits
hemoglobin molecule: 10^{4} bits
simple molecules 10^{2} bits
atom (two-level): 1 bit

Figure 12.1 Emergent Levels of Complexity. At bottom the most elementary atomic structure is the 2-level atom with 1 bit states. At top is the accessible universe with 10^{90} bits.

Propagation of New Information into Classical Scales

Entangled automata of multiple atoms and molecules with abundant degrees of freedom similarly produce entanglement and dis-entangling operations in the natural execution of observation/evolution (Posulates II and III). Large numbers of degrees of freedom released in a gas are maximally random. But ordered into molecules, and supermolecules (most common in organic molecules such as DNA, proteins, lipids, and others), the extensive random motion of these freedoms is quenched in condensation, chemically bonded, and repurposed as suprarmolecular structures[31]. This process is fundamental to the order and structure of the universe.

Principle (Order and Condensed Matter)**.** *The condensation of matter with the quench of random degrees of freedom is an engine of order-making in the universe.*

Emergent and entangled molecular superstructures possess an autonomy of their own. Their interaction with the small molecules of liquid or air environments leads to decoherence of quantum superpositions of these quasi-macroscopic structures. [7, 13, 14]. Because of the large difference between the massive particle and the small molecules of an air or water environment, this system can be described by dividing the totality into a system and environment (Theory of Open Quantum Systems), a useful procedure for the analyst. Autonomous Quantum Theory naturally incorporates such procedures and extends well beyond to include nature where human analysts are not present.

The detailed process by which atoms and molecules are repurposed to complex ordered structures by a cascade of chemical binding beginning with simple atoms and proceeding in complexity to supramolecules is not presently well-established. The state of understanding is similar to (and deeply related to) that of the emergence of the first biological cell from pre-biotic chemicals and chemical cycles.

The step-up from the qubit or the qubit pair automaton to general states and dynamics of nature is a very great one. Agents observing and acting with one another pack a combinatorial explosion of information. If an overload of information swamps human observers, does it not also swamp individual quantum agents in their observation dynamics of such vast numbers of other agents?

Nature manages this information explosion in one of the most magnificent—and inauspicious—dramas of the physical world: the transformation of the gas phase of the primal big-bang and the creation of the heavy elements in the burning of stars and the formation of condensed matter. The 10^{90} qubits naturally segregate into organized emergent levels roughly shown in Figure 12-1. The feature of nature that brings order to this explosion is the condensation of matter into phases and their phase boundaries. Who would have thought that purely quantum-mechanical van der Walls forces between atoms (which vary as the inverse-seventh power of the separation between them) are responsible for the initial waves of formation of condensed structures. These condensed structures in turn absorb the explosive randomness of the primal gas phase, bringing in new order at higher emergent levels

Autonomous Quantum Theory as a Theory of Nature

Quantum Theory in the twenty-first century is strongly human-centric. It is concerned with the kinds of measurements human beings can perform, the kinds of quantum processes they can conceive and manipulate, and the kinds of things they can create—with quantum computers being a central objective. I have explored a far larger natural world. *Autonomous Quanta* is directed to the operation of nature as a whole in an attempt to answer the question, "how does nature manage her dynamics generally, independently of human beings?"

Autonomous Quantum Theory offers a challenging and untested portrait of quantum nature. Perhaps the most controversial, beyond the postulate of agency, are Postulates II and III which take the Wheeler and Peres Principles deeply seriously: *the dynamics of a quantum agent are directed by the Hamiltonian observed by that agent.* A definitive experiment that showed this was not the case would throw this theory into question.*

*For example if the motion of a quantum agent (a single degree of freedom) was controlled by a Hamiltonian omnisciently available to all relevant freedoms.

If Postulate III were shown to be invalid, what is the way to a more compelling theory? The necessary role of agency and observation under an Uncertainty Principle seems inescapable. Any future theory must come to grips with that reality. Wheeler's musing "how equations might fly" and, more pointedly, how dead birds thrown into the air follow a parabola while live birds go where they will, remains a basic challenge for any theory of nature.

Notes

[1] ‘Action’ S or the third Pauli operator has an interesting history. Pauli [W. Pauli in *Handbuch der Physik*, H. Geiger and K. Scheel (eds), (Springer, Berlin, 1933), Vol 34, Pt. 1, p. 98.] asked if the position and momentum operators were sufficient to fully describe the statistics of a quantum particle. He was considering a particle moving on the continuum of the real line, not the finite qubit I am considering. In that case, action, as the product of position and momentum has a continuous limit as the identity with the Planck quantum as a coefficient. Not so for the qubit which is “the most quantum” of systems with two independent agents but only one bit of definite information; the other bit random. If observed values, $(\mathsf{q} \parallel \mathsf{p})$, followed the unitary algebra, statistics of the position-momentum pair would provide a complete account of a quantum particle including the statistics the third agent, the action agent. For example it would be true that $(\mathsf{s} \parallel \mathsf{q}) = i(\mathsf{p} \parallel \mathsf{q})(\mathsf{q} \parallel \mathsf{q})$ which is false. (See Eq. (7.5).)

[2]Experiments aimed at determining the state of the agent community are done by preparing a large number of qubits in an identical known state (say, ‘on’) and running them (individually) through a detector array which pops out one or more of the agent-states as definite output. These qubits could be an atoms in a double well in which the position agent is definitely in the first well, for example. To set the value of another agent, say the momentum agent, you must impose a velocity on that atom sitting in the first well.

Immediately, there is a problem. Only 1 bit of the qubit can be given a definite value. If it has been given to the position agent, no information is left for the others. Try as you will, you will not be able to also give definite values to the momentum or action agents. The Uncertainty Principle forbids it. (You could alternatively impose a definite velocity on the atom—thereby fixing its momentum agent that now definitely cycles the atom continuously through both wells. But now it is impossible to obtain a definite fix on the position of which well the the atom resides.)

You will not be surprised to learn that these experiments done with the same identical preparation in which 1 bit of definite information is fixed in one of the three agents of the qubit do not in general give identical, repeatable results when all three agents are measured or observed. Like the qubit light switch, they are laced with randomness. Indeed, they must be because the qubit can at most pack 1 bit of definite information and the agent community of the automaton carries 2 bits of information.

If the experiment is set to measure one of the agents definitely and the prepared copies of this agents are all in the ‘on’ state, that agent will pop out a definite stream of ‘on’s. But the other two agents measured simultaneously give results that are randomly off and on. And if the experiment is set up to privilege none of the agents in definiteness , all $2^3 = 8$ joint states will emerge randomly with equal probabilities of 1/8. Adding the results up for the 8 agent-states multiplied by their probabilities of 1/8, one gets a total information count of 1 bit—the definite capacity of the qubit. 1 bit prepared, 1 bit emerges definitely, 1 bit emerges randomly.

This experiment is the basis for the quantum generation of random bit strings. Should you like one, you can go to www.hotbits [16] and download a random bit string corresponding to the indefinite agents created by the process I have just described. (Sinless randomness in contrast to von Neumann’s “sinful” randomness[†]

[†]John von Neumann was known for his characterization of those who believe in the possibility of producing random digits arithmetically as being “in a state of sin.” Random digits produced by nature avoid arithmetic means.

Appendix A

Number of Real Numbers to Specify a Quantum State

A general $d \times d$ complex matrix of full rank is specified by d^2 complex parameters or $2d^2$ real parameters. The number of real parameters fixing the diagonal entries is $2d$ and the number of real parameters fixing the off-diagonal entries is $2d(d-1)$. For $d = 2$ these numbers are 8 real parameters for the full matrix, 4 real parameters for the diagonal and 4 real parameters for the off-diagonal entries.

Hermitean matrix entries satisfy the condition that transposes are complex conjugates: $a_{jk} = a^*_{kj}$. The number of real parameters fixing the skew entries of a $d \times d$ Hermitean matrix is $d(d-1)/2$. The number of entries on the diagonal (all real) is d for or a total of $d(d-1) + d = d^2$ real parameters—half the number of a general complex matrix.

A $d \times d$ Hermitean matrix of rank r has r real eigenvalues and eigenspaces. Each eigenspace bearing a real eigenvalue is a line defined by $d - 1$ complex parameters (or $2(d-1)$ real parameters). Since the lines are orthogonal, it takes $2(d-1)$ real parameters to specify the first eigenspace, $2(d-2)$ for the second, down to $2(d-r)$ for the last. The number of parameters that fix the eigenspaces is therefore

$$2\left[(d-1) + (d-2) + \cdots + (d-r)\right] = 2\left[dr - \frac{r(r+1)}{2}\right] = 2dr - r^2 - r.$$

Adding in the r real parameters that specify the eigenvalues there results

$$\text{Number of real parameters} = r + 2dr - r^2 - r = 2dr - r^2.$$

A Hermitean matrix of full rank $r = d$ is specified by d^2 real parameters. At the other extreme, a rank $r = 1$ Hermitean matrix is specified by $2d - 1$ real parameters.

A Hermitean matrix serving as a density matrix must be normalized; it is specified by $d^2 - 1$ real parameters at full rank. The number of parameters to fix a rank $r = 1$, density matrix is $2(d - 1)$.

For a Hermitean matrix of the qubit automaton ($d = 2$), 4 real numbers are required to fix the state at full rank. For a normalized Hermitean matrix (such as a density matrix) of full rank, this number is 3.

Appendix B

Unitarity Algebra of Multiple Qubits

Hermitian operators—the observables of quantum nature—are simultaneously the generators of unitary transformations—transformations that map one set of observables to another preserving their Hermitian (inner product) norm.

$$\mathsf{U}(a) = e^{ia_\mu \boldsymbol{\sigma}_\mu}.$$

The amplitudes of the map a_μ describe the intervals between the states the unitary acts on.

The four operators for each qubit in a system of N qubits that manifest its action capacity are

$$\boldsymbol{\sigma}^k_\mu, \quad \mu = 1, 2, 3, 4, \qquad k = 1, 2, \ldots, N.$$

Quantum automata manifest *unitary invariance*in the $SU(2)$ unitary algebra they obey. The invariant quantity is the inner product of Hilbert space state-vectors or the trace of Hilbert-Schmidt operators. This quantity is invariant as automata evolve new states under a unitary operator U.

$$\boldsymbol{\sigma}' \leftarrow \mathsf{U}^\dagger \boldsymbol{\sigma} \mathsf{U}$$

$$\mathsf{Tr}\,(\boldsymbol{\sigma}') = \mathsf{Tr}\,(\boldsymbol{\sigma})\,.$$

$$\mathsf{U}^\dagger \mathsf{U} = \mathsf{I}.$$

Operators of different qubits commute:

$$[\boldsymbol{\sigma}^k_\mu \boldsymbol{\sigma}^l_\nu] = 0, \quad k \neq l, \quad \mu, \nu, \lambda = 1, 2, 3, 4. \tag{B.1}$$

Operators for the same qubit cyclically produce the third—the group algebra product

$$\boldsymbol{\sigma}^k_\mu \boldsymbol{\sigma}^k_\nu = \epsilon_{\mu\nu\lambda} i \boldsymbol{\sigma}^k_\lambda, \quad \mu, \nu, \lambda = 1, 2, 3., \qquad k = 1, 2, \ldots, N. \tag{B.2}$$

Identical operators acting on one another produce the identity,

$$\boldsymbol{\sigma}_\mu^k \boldsymbol{\sigma}_\mu^k = {\boldsymbol{\sigma}_\mu^k}^2 = \mathsf{I}, \quad \mu = 1, 2, 3., \qquad k = 1, 2, \ldots, N. \tag{B.3}$$

The identity, the "self" or logical agent is labelled as

$$\boldsymbol{\sigma}_4^\mu = \mathsf{I}$$

and completes the description.

Appendix C

Canonical Description of Multi-Qubit Automata

A quantum system of many freedoms in computational representation takes form as a tensor product of qubits. Each degree of freedom bears a pair of independent agents. A system of N qubits indexed by $k = 1, 2, \ldots, N$ has $2N$ independent agents.

Canonical Supercoordinates

The agent description in operator language $\boldsymbol{\sigma}^{(k)}$ has an equivalent description in canonical coordinate language $\mathsf{Q}^{(k)}, \mathsf{P}^{(k)}$ (in agent-actor presentation) and $\mathsf{q}^{(k)}, \mathsf{p}^{(k)}$ (in agent-state presentation). The canonical pair are independent coordinates which, according to the unitary algebra, generate a third, but dependent coordinate: $\mathsf{S}^{(k)}$ and $\mathsf{s}^{(k)}$ in agent-actor and agent-sate presentation respectively. The agent-actor presentation translates as

$$\mathsf{Q}^{(k)} \equiv \boldsymbol{\sigma}_3^{(k)}, \qquad \mathsf{P}^{(k)} \equiv \boldsymbol{\sigma}_1^{(k)}, \qquad \mathsf{S}^{(k)} \equiv \boldsymbol{\sigma}_2^{(k)},$$

while that for agent-states reads

$$\mathsf{q}^{(k)} \equiv \frac{1}{2}(\mathsf{I} + \mathsf{Q}^{(k)}), \quad \mathsf{p}^{(k)} \equiv \frac{1}{2}(\mathsf{I} + \mathsf{P}^{(k)}), \quad \mathsf{s}^{(k)} \equiv \frac{1}{2}(\mathsf{I} + \mathsf{S}^{(k)}).$$

The agents of a multi-qubit system may now be cast in canonical form in terms of their canonical agent-state supercoordinates . These are N-length vectors whose entries are $2^N \times 2^N$ tensor products of the canonical coordinates (in either agent-sate q, p or agent-actor Q, P presentations respectively) and the identity for each qubit. Each of these entries is a $2^N \times 2^N$ matrix where the particular position coordinate of a given qubit lies on the diagonal entry tensored with the other $N-1$ entries as 2×2 identities I. For example, the agent-actor supercoordinates are

$$\mathbf{Q}=\begin{bmatrix}\mathsf{Q}^{(1)}\mathsf{I}\mathsf{I}\cdots\mathsf{I}\\ \mathsf{I}\mathsf{Q}^{(2)}\mathsf{I}\cdots\mathsf{I}\\ \vdots\\ \mathsf{I}\mathsf{I}\cdots\mathsf{I}\mathsf{Q}^{(N)}\end{bmatrix}\qquad \mathbf{P}=\begin{bmatrix}\mathsf{P}^{(1)}\mathsf{I}\mathsf{I}\cdots\mathsf{I}\\ \mathsf{I}\mathsf{P}^{(2)}\mathsf{I}\cdots\mathsf{I}\\ \vdots\\ \mathsf{I}\mathsf{I}\cdots\mathsf{I}\mathsf{P}^{(N)}\end{bmatrix}$$

The corresponding agent-state supercoordinates are *

$$\boldsymbol{q}=\frac{1}{2^N}(\mathsf{I}_{2^N}+\mathbf{Q}),\qquad \mathbf{p}=\frac{1}{2^N}(\mathsf{I}_{2^N}+\mathbf{P}),$$

The dependent agent coordinates are also N-vectors which in terms of the canonical pairs are

$$\mathbf{S}=i\mathbf{P}\cdot\mathbf{Q}\qquad \boldsymbol{s}=i[\boldsymbol{q},\mathbf{p}]+\frac{1}{2^N}\mathsf{I}_{2^N}.$$

In summary, an N-freedom quantum system is represented by a pair of independent N-vector supercoordinates $\boldsymbol{q},\mathbf{p}$ each component of which is an agent

$$\boldsymbol{q}=\boldsymbol{q}(\boldsymbol{q}^{(1)},\boldsymbol{q}^{(2)},\ldots,\boldsymbol{q}^{(N)})\qquad \mathbf{p}=\mathbf{p}(\mathbf{p}^{(1)},\mathbf{p}^{(2)},\ldots,\mathbf{p}^{(N)})$$

and a dependent N-vector supercoordinate given in terms of them by

$$\boldsymbol{s}=i[\boldsymbol{q},\mathbf{p}]+\frac{1}{2^N}\mathsf{I}_{2^N}.$$

Supercoordinate Tensor Chemistry

Hamiltonian in Supercoordinates

The canonical coordinate agents are completed by the global Hamiltonian agent. The Hamiltonian as agent-actor or agent-state in the N-freedom setting are $2^N\times 2^N$ tensor product operators

$$\mathbf{H}=\mathbf{H}(\boldsymbol{q},\mathbf{p}),\qquad \mathbf{h}=\frac{1}{2^N}(\mathsf{I}_{2^N}+\mathbf{H}).$$

*Written out in terms of individual qubit agents these are

$$\boldsymbol{q}=\begin{pmatrix}\mathsf{q}^{(1)}\mathsf{I}\mathsf{I}\cdots\mathsf{I}\\ \mathsf{I}\mathsf{q}^{(2)}\mathsf{I}\cdots\mathsf{I}\\ \vdots\\ \mathsf{I}\mathsf{I}\cdots\mathsf{I}\mathsf{q}^{(N)}\end{pmatrix}\qquad \mathbf{p}=\begin{pmatrix}\mathsf{p}^{(1)}\mathsf{I}\mathsf{I}\cdots\mathsf{I}\\ \mathsf{I}\mathsf{p}^{(2)}\mathsf{I}\cdots\mathsf{I}\\ \vdots\\ \mathsf{I}\mathsf{I}\cdots\mathsf{I}\mathsf{p}^{(N)}\end{pmatrix}$$

Canonical Unitary Transformations

The coordinate language describing a quantum system is not unique. The description of a quantum system in coordinates $\boldsymbol{q}, \mathbf{p}$ with Hamiltonian $\mathbf{h}(\boldsymbol{q}, \mathbf{p})$ is physically equivalent to that in language $\boldsymbol{q}', \mathbf{p}'$ with Hamiltonian $\mathbf{h}'(\boldsymbol{q}', \mathbf{p}')$ where the two sets are related by a canonical unitary transformation U

$$\boldsymbol{q}' = \mathsf{U}^\dagger \boldsymbol{q} \mathsf{U}, \qquad \mathbf{p}' = \mathsf{U}^\dagger \mathbf{p} \mathsf{U},$$

where the unitaries satisfy

$$\mathsf{U}^\dagger \mathsf{U} = \mathsf{I}.$$

The Hamiltonian state is invariant in both languages,

$$\mathbf{h}(\boldsymbol{q}, \mathbf{p}) = \mathbf{h}'(\boldsymbol{q}', \mathbf{p}')$$

which specifies the Hamiltonian actor in both coordinate languages,

$$\mathbf{H}(\boldsymbol{q}, \mathbf{p}) = 2\mathsf{h}(\boldsymbol{q}, \mathbf{p}) - \mathbf{I} = \mathbf{H}'(\boldsymbol{q}', \mathbf{p}') = 2\mathbf{h}'(\boldsymbol{q}', \mathbf{p}') - \mathbf{I}.$$

The canonical character of the transformation renders the equations of motion the same in both languages,

$$\frac{d\boldsymbol{q}'}{dt} = (i/\hbar)[\boldsymbol{q}', \mathbf{H}'(\boldsymbol{q}', \mathbf{p}'], \quad \frac{d\mathbf{p}'}{dt} = (i/\hbar)[\mathbf{p}', \mathbf{H}'(\boldsymbol{q}', \mathbf{p}'].$$

Since the Identity makes no contribution to the commutator, equations of motion are the same up to a factor of 2 whether the Hamiltonian appears as agent-actor $\mathbf{H}$ or as agent-state $\mathbf{h}$:

$$\frac{d\boldsymbol{q}'}{dt} = (2i/\hbar)[\boldsymbol{q}', \mathbf{h}'(\boldsymbol{q}', \mathbf{p}'], \quad \frac{d\mathbf{p}'}{dt} = (2i/\hbar)[\mathbf{p}', \mathbf{h}'(\boldsymbol{q}', \mathbf{p}'].$$

Observations

An observation of the Hamiltonian by a canonical coordinate agent produces an observed agent-state, one for each of the $\boldsymbol{q}$s and one for each of the $\mathbf{p}$s but rendered in the projectors of the observer rather than those of the Hamiltonian. The $2N$ independent observations fill two N-length vectors, each component of which is a $2^N \times 2^N$ operator like the canonical agent observed, as indeed they must be: independent observations must have a count equal to the number of independent canonical coordinates

$$(\mathbf{h} \parallel \boldsymbol{q}) = \begin{pmatrix} (\mathsf{h} \parallel \boldsymbol{q}^{(1)}) \\ (\mathsf{h} \parallel \boldsymbol{q}^{(2)}) \\ \vdots \\ (\mathsf{h} \parallel \boldsymbol{q}^{(N)}) \end{pmatrix}, \qquad (\mathbf{h} \parallel \mathbf{p}) = \begin{pmatrix} (\mathsf{h} \parallel \mathbf{p}^{(1)}) \\ (\mathsf{h} \parallel \mathbf{p}^{(2)}) \\ \vdots \\ (\mathsf{h} \parallel \mathbf{p}^{(N)}) \end{pmatrix},$$

The four independent agent-states of the qubit pair are:

$$\boldsymbol{q} = \begin{pmatrix} \mathsf{q}^{(1)}\mathbf{i} \\ \mathbf{i}\mathsf{q}^{(2)} \end{pmatrix} \qquad \mathbf{p} = \begin{pmatrix} \mathsf{p}^{(1)}\mathbf{i} \\ \mathbf{i}\mathsf{p}^{(2)} \end{pmatrix}$$

The four independent observables of a qubit pair are:

$$(\mathbf{h} \parallel \boldsymbol{q}) = \begin{pmatrix} (\mathsf{h} \parallel \mathsf{q}^{(1)}\mathbf{i}) \\ (\mathsf{h} \parallel \mathbf{i}\mathsf{q}^{(2)}) \end{pmatrix}, \qquad (\mathsf{h} \parallel \mathbf{p}) = \begin{pmatrix} (\mathsf{h} \parallel \mathsf{q}^{(1)}\mathbf{i}) \\ (\mathsf{h} \parallel \mathbf{i}\mathsf{q}^{(2)}) \end{pmatrix}.$$

Dynamics

Supercoordinate evolution is directed by the agent-observed Hamiltonians $(\mathsf{H} \parallel \boldsymbol{q}(t)), (\mathsf{H} \parallel \mathbf{p}(t))$:

$$\boldsymbol{q}(t') = e^{-(i/\hbar)(\mathsf{H}\,\|\,\boldsymbol{q}(t))(t'-t)}\boldsymbol{q}(t)e^{(i/\hbar)(\mathbf{H}\,\|\,\boldsymbol{q}(t))(t'-t)},$$

$$\mathbf{p}(t') = e^{-(i/\hbar)(\mathsf{H}\,\|\,\mathbf{p}(t))(t'-t)}\mathbf{p}(t)e^{(i/\hbar)(\mathbf{H}\,\|\,\mathbf{p}(t))(t'-t)}.$$

Appendix D

Theory of Probabilistic Observation

The probability density $\mathfrak{p}_{\mathsf{a}k}$ is the probability agent a whose states range over a space of $k = 1, 2, \ldots d$ elements is in the state $|\mathsf{a}, k\rangle$ satisfying

$$\sum_k \mathfrak{p}_{\mathsf{a}k} = 1, \qquad k = 1, 2, \ldots, d., \quad \mathsf{a} = 1, 2, \ldots, d^2.$$

The density matrix of agent a in its self-frame is

$$\mathsf{a} = \sum_k |\mathsf{a}, k\rangle \mathfrak{p}_{\mathsf{a}k} \langle \mathsf{a}, k|.$$

Agent b with density matrix $\mathsf{b} = \sum_j |\mathsf{b}, j\rangle \mathfrak{p}_{\mathsf{b}j} |\mathsf{b}, j\rangle$ in its self-frame 'measures' the state a. The result is $(\mathsf{a} \parallel \mathsf{b})$: the state a as it appears in the frame of agent b:

$$(\mathsf{a} \parallel \mathsf{b}) = \sum_j |\mathsf{b}, j\rangle \sum_k \langle \mathsf{b}, j|a, k\rangle \mathfrak{p}_{\mathsf{a},k} \langle \mathsf{a}, k|\mathsf{b}, j\rangle \langle \mathsf{b}, j|,$$

The transition probability

$$\mathfrak{P}_{p,j|\mathsf{a},k} = |\langle \mathsf{b}, j|\mathsf{a}, k\rangle|^2,$$

is a conditional probability for the state $|\mathsf{b}, j\rangle$ given the state $|\mathsf{a}, k\rangle$. Then the Bayes theorem yields the probability that agent b holds the state $|\mathsf{b}, j\rangle$ given transitions from all the states $k = 1, 2, \ldots, d$ of a: the conditional transition probability for $|\mathsf{b}, j\rangle$ given a is thet sum of the contributions from all states of a,

$$\mathfrak{p}_{\mathsf{b}j|\mathsf{a}} = \sum_k \mathfrak{P}_{\mathsf{b}j|\mathsf{a}k} \mathfrak{p}_{\mathsf{a}k} = \sum_k |\langle \mathsf{b}, j|\mathsf{a}, k\rangle|^2 \mathfrak{p}_{\mathsf{a}k}.$$

The resulting observation of the state of a in the frame of b is then

$$(\mathsf{a} \parallel \mathsf{b}) = \sum_j |\mathsf{b}, j\rangle \mathfrak{p}_{\mathsf{b}j|\mathsf{a}} \langle \mathsf{b}, j|.$$

To overemphasize a point, the self-observation of an agent represents its state in its own frame of reference but an observation of that agent by a different agent is manifest in the observing agent's frame. Note that the trace is preserved in observation of agent-states: the observed agent state has unit trace as does the agent state observed;

$$\mathsf{Tr}\left((\mathsf{a} \parallel \mathsf{b})\right) = 1.$$

The conjugate observation of b by a is

$$(\mathsf{b} \parallel \mathsf{a}) = \sum_k |\mathsf{a}, k\rangle \mathfrak{p}_{\mathsf{a}k|\mathsf{b}} \langle \mathsf{a}, k|,$$

with

$$\mathfrak{p}_{\mathsf{a}k|\mathsf{b}} = \sum_j \mathfrak{P}_{\mathsf{a}k\mathsf{b}j} \mathfrak{p}_{\mathsf{b}j|\mathsf{a}}.$$

Note that the transition probabilities between states of the same agent—the self-observation—is

$$\mathfrak{P}_{\mathsf{a}k,\mathsf{b}j} = |\langle \mathsf{a}, j|\mathsf{a}, k\rangle|^2 = \delta_{jk}$$

and

$$\mathfrak{p}_{aj|a} = \sum_k \mathfrak{P}_{aj|ak} \mathfrak{p}_{ak} = \mathfrak{p}_{aj}.$$

Self-observation reproduces the information state of the agent

$$(\mathsf{a} \parallel \mathsf{a}) = \sum_j |a, j\rangle \mathfrak{p}_{aj} \langle a, j|.$$

Principle (Evolution of the Agent-State). *The evolution of agent-states cannot, in general, be unitary because the observed Hamiltonian is not in general definite. This means agents, other than the Hamiltonian superagent, find themselves in new states that are not necessarily unitarily evolved from their previous states. The result: a statistical theory driven by agent-observed Hamiltonians. Agent holds its current state and the "script" of the Hamiltonian code it has imperfectly observed from the Hamiltonian superagent. From this, the time-evolution operator executes the agent's motion to its new state.*

If the agents are complementary so $\mathfrak{P}_{b,j|a,k} = |\langle b, j|a, k\rangle|^2 = 1/d$, $\forall \{a, b, j, k\}$, then the conditional probability is uniform and

$$\mathfrak{p}_{bj|a} = \frac{1}{d}$$

The observed state of a by b, $(a \parallel b)$ is a completely mixed state *independent of the probability distribution* $\mathfrak{p}_a$ on a.

$$(a \parallel b) = \frac{1}{d}\mathsf{I}.$$

The maximally uncertain nature of b's observation of a is a consequence of their complementarity and not the uncertainty or mixedness in the self-observed state of a itself.

If the agents a, a' are different agents of the same species they have the same eigenvectors but different eigenvalues. Suppose a' observes a. The transition probability becomes $\mathfrak{P}_{aj|a'k} = \delta_{jk}\delta_{aa'}$ and the probability for a' observing a becomes

$$\mathfrak{p}_{a'j|a} = \sum_k \mathfrak{P}_{a'j|ak}\mathfrak{p}_{ak} = \sum_k \delta_{jk}\delta_{aa'}\mathfrak{p}_{ak} = \mathfrak{p}_{aj}\delta_{aa'}.$$

An agent a' therefore faithfully observes another agent a of the same species as their eigenstates are identical: $|\mathsf{a}', j\rangle = |\mathsf{a}, j\rangle$.:

$$(\mathsf{a} \parallel \mathsf{a}') = \sum_j |a', j\rangle \mathfrak{p}_{aj}\delta_{aa'}\langle a', j| = \mathsf{a}$$

Transitions to different agents in the same complement vanish. Only the self-transition survives with the corresponding probability for that agent showing the fixed point property

$$(\mathsf{a} \parallel a') = (\mathsf{a} \parallel \mathsf{a}) = \mathsf{a}.$$

This says that the interaction of two agents in the same complement vanishes save for the self-interaction which is a transition to the same agent with the same probability distribution.

Appendix E

Additional Thoughts on the Relationships Between Algorithmic and Quantum Randomness

The Gödel and Turing perspectives of uncomputability and randomness are now historical. Recently, F. William Lawvere [25] has elegantly provided an alternative perspective which still turns on the mis-match between the countable and uncountable, but the formulation improves on economy and transparency over that of Gödel. In particular, the connection between uncountable sets and self-reference is made explicit. Like the set of theorems in the 'historical' view of Godel incompleteness (which turns on the size of the set of theorems T compared to that of axioms Y), the maps $T \to Y$ are uncountable.

The set of maps from a set T to set Y is Y^T and is exponentially larger than either of the sets T or Y for cardinalities greater than 1. . The maps from sets to *maps of sets*

$$f : T \to Y^T$$

are equivalent to maps of the Cartesian product of theorems to the set of axioms Y [29]:

$$\hat{f} : T \times T \to Y. \tag{E.1}$$

The functions $\hat{f}$ may be converted to equivalent functions f of two arguments

$$f(t, t') = \hat{f}(t')t.$$

Functions of *maps of sets* to the sets they map can be seen to have the capacity for self-reference. They require two copies of the set being mapped. Functions $\hat{f} : T \times T \to Y$ exhibited in Eq. (E.1) make it possible for their arguments to "talk about themselves." At the same time, self-reference makes such functions uncomputable as I shall shortly illustrate.

A computable string of bits is understood to be definite: it can be exhibited in a memory location of a Turing machine. An uncomputable string cannot; it is incommensurate with countable memory, "infinitely outstripping it." The Turing machine reveals this limitation by not accommodating a program P that decides if a string given as input halts with a result or does not. The culprit is self-reference which can be seen in this simple proof of the impossibility that such a program exists.

Principle (Self-Reference and Uncomputability)**.** *The proof that the maps* (E.1) *are uncomputable proceeds by assuming a program P exists that computes them and then by showing that assumption leads to a contradiction. Embed P in another program P' that takes programs Q as input and runs Q's own code with the condition: (1) run forever if Q halts or (2) halt if Q runs forever. The proof is completed by running P' with its own code as input. Then P' halts if it runs forever and runs forever if it halts: contradiction. The assumption that P' and hence P exist is false.*

The self-reference gremlin shows itself in the need for a copy of P's own code for it to run as input to itself to make the proof viable, "the program eats its own code." In a classical universe, copies may be freely assumed. This is not possible in a quantum universe as I shall now explain.

Algorithmic Randomness Meets the Quantum

Several authors have explored the relationship between quantum and algorithmic randomness (see [38, 40] and [39] and the references therein).

Quantum randomness is both mathematical *and* physical, both software, in probability distributions , and hardware, in physical particles (such as electrons and atoms translating and spinning that support those probabilities.) Algorithmic randomness is a creature of pure computation without physical manifestation. Turing machines exist in a platonic mathematical universe which may or may not be manifested in nature.

Two impediments prevent algorithmic randomness a definitive role in quantum dynamics: (1) a *No-Cloning Theorem* which undermines bellwether proofs, such as Gödel's Incompleteness Theorem, which rely on introducing a program's self-description as input and (2) An incapacity to describe randomness in countable assemblies of quanta (as the Heisenberg Principle readily does). I'll take up the No-Cloning Principle first.

Principle (No-Cloning Theorem)**.** *Assume a cloning operator* C *exists which takes* *a quantum state* $|\mathsf{q}\rangle$ *and an arbitrary state* $|\mathsf{q}'\rangle$ *which it is to transform into a copy of* $|\mathsf{q}\rangle$*:*

$$\mathsf{C}|\mathsf{q}\rangle|\mathsf{q}'\rangle \rightarrow |\mathsf{q}\rangle|\mathsf{q}\rangle.$$

It is sufficient to demonstrate this process for for a single qubit,

$$|\mathsf{q}\rangle = \alpha|0\rangle + \beta|1\rangle,$$

for which it follows

$$\begin{aligned} \mathsf{C}|\mathsf{q}\rangle|\mathsf{q}'\rangle \rightarrow &\quad (\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle) \\ \rightarrow &\quad \alpha^2|0\rangle|0\rangle + \alpha\beta|0\rangle|1\rangle + \beta\alpha|1\rangle|0\rangle + \beta^2|1\rangle|1\rangle. \end{aligned} \tag{E.2}$$

This putative clone is not linear in the amplitudes α, β *but quadratic. No linear unitary operator can execute such an operation. Hence the assumed cloning operator does not exist.*

The impossibility of state copying invalidates the self-reference fundamental to the uncomputabiity and incompleteness proofs typified by (E.1) in a quantum world.

Infinity and Algorithmic Randomness

Quantum physics rests on *both* physical and mathematical foundations. The Heisenberg Principle demonstrates the randomness in nature without any appeal to uncountable cardinalities. The qubit automaton (which maps qubit states to qubit states) requires 2 bits to specify its state. But the Heisenberg principle requires that only one of these bits can be specified or measured. The other, upon measurement, will produce a random result. All of this is in a finite context.

Principle (A finite or countably infinite universe is not random in the sense of Algorithmic Randomness)**.** *Because Algorithmic Randomness rests on the existence of uncountably infinite sets of elementary particles or bits, a countably infinite, but not necessarily uncountable, universe is not algorithmically random.*

Appendix F

Autonomous Quantum Theory and the Bayesian Interpretation

Probability theory is well-founded with respect to human agents. (Indeed, much of the origins of the subject with Laplace, Craig, etc. emerged from questions about gambling.) The gambling origin points out the role of counting cards and die faces, etc. in guiding betting. But in real, human multiplayer games, there is a psychology of body posture, facial expressions, and other behavioral idiosyncrasies essential to bluffing which poker players tell us is quite important. Deception is also a part of animal communication and behavior generally.

Against this background, the procedure of making "rational bets" in the framework of Bayesian reasoning as a theory of quantum mechanics seems to me too tied to human agents. The earth, let alone the universe, has existed and executed its quantum dynamics without human beings—or even higher animals—for most of its history. That said, it is possible, in my judgement, for a rich enough assembly of automata to reason somewhat in Bayesian fashion, given its semi-definite roots in determinism. But, ultimately, a quantum system, will always be subject to elements of quantum randomness "from the outside" which give it the spark of creativity and a life-like quality.

Appendix G

The Particle Wave

The continuous beam of particles passing through the double slit can be represented as a qubit where each of its two states is a continuous function of a position coordinate, one for the path throiugh each slit, x_1 and x_2. Represent the beam's agent-states as position agents, $|\mathsf{q},1\rangle(x_1)$ and $|\mathsf{q},2\rangle(x_2)$: As an example, if the beams are plane waves with wave number k and frequency ω, the beam pure position state-vectors are

$$|\mathsf{q},1\rangle(x) = e^{i(kx_1-\omega t)}\begin{pmatrix}1\\0\end{pmatrix}, \qquad |\mathsf{q},2\rangle(x) = e^{i(kx_2-\omega t)}\begin{pmatrix}0\\1\end{pmatrix}.$$

These beam states are orthogonal and complete

$$\langle \mathsf{q},j|\mathsf{q},k\rangle = \delta_{jk}, \qquad \sum_j |\mathsf{q},j\rangle\langle\mathsf{q},j| = \mathsf{I}.$$

The agent state exiting the double slit is an equal superposition "superposition agent" composed of particle states at the two slits:

$$|\mathsf{q}\rangle(x_1,x_2) = \frac{1}{\sqrt{2}}\big(|\mathsf{q},1\rangle(x_1) + |\mathsf{q},2\rangle(x_2)\big) = \frac{1}{\sqrt{2}}e^{-i\omega t}\begin{pmatrix}e^{ikx_1}\\e^{ikx_2}\end{pmatrix}.$$

The corresponding density matrix of this agent-state incident on the double slit is

$$\mathsf{q}(x_1,x_2) = \tfrac{1}{2}\big(|\mathsf{q},1\rangle(x_1) + |\mathsf{q},2\rangle(x_2)\big)\big(\langle\mathsf{q},1|(x_1) + \langle\mathsf{q},2|(x_2)\big) \tag{G.1}$$

$$= \tfrac{1}{2}\begin{pmatrix}1 & e^{ik(x_2-x_1)}\\ e^{-ik(x_2-x_1)} & 1\end{pmatrix} \tag{G.2}$$

Appendix H

The Duality of Quantum Information and Matter

Aristotle had a sense of nature that prefigured quantum discoveries. He proposed that all the objects in nature were composed of two ingredients—prime matter and substantial form. Prime matter was material stuff, but it was infinitely fungible. It wasn't any particular thing but had the potential to be any material thing whatsoever: some-thing. A mound of sculptor's clays was surely his model/metaphor. Substantial form (applied by the sculptor in the metaphor) endowed prime matter with that some-thingness.

Aristotle emphasized that the pair were inseparable. One did not find pure matter or pure form cleaved from one another. Form was embodied in matter. Matter was embodied form. (Subsequently, Aristotle and, most notably, Thomas Aquinas attempted to argue that substantial form could exist without matter. They were, of course, trying to establish the reality of the soul, and particularly, the immortality of the soul. I do not find those immortality arguments compelling becuase I find the Landauer principle deeply fundamental. I'm quite happy with the mortal soul.)

Aristotle's prime matter lines up with our modern notion of mass-energy (recall that since Einstein, mass and energy are different forms of the same thing *mass-energy*). Aristotle's substantial form lines up with the modern notion of information. Information is the data contained in strings of bits. The bits aren't the matter. The bits are matter's information content. But the matter is not the bits either. The matter is manifest in mass, charge, spin, and other physical properties. The particular values of those properties can be in superposition which constitutes the wave function, or in modern quantum theory, Heisenberg's operators—the agents of Quantum Automata.

The ”wave function” was an entity distinct from the positions and momenta of particles. It was suddenly the new kid on the block. The wave function nicely completed the missing element from the duality: the missing information.

Bits don't reside in airy nothingness. Bits are encoded in matter from the toggle of a light switch to the positions of nucleotides in a DNA molecules. it isn't until bits are given values that an information string becomes *some-thing.* And when the bits are changed, that something becomes some-thing else. In the modern idiom of computer science, Aristotelian prime matter is *hardware*, substantial form, *software.* Hardware can't compute without software. Software has no existence outside hardware. Even non-electronic software such as code printed or hand-written on a page is bound to the hardware of paper and ink.

The Aristotelian principle of matter and form with *matter* understood as atoms, molecules and other elementary particles and *form* as the information that specifies the state of these particles is fundamental to natural computation.To illustrate: A lipid molecule consists of approximately 100 C, N, O, H, and P atoms. That is the lipid matter in mass-energy. These atoms are formless until bonded together in some molecular structure—including the possibility they may be dissociated as simple atoms at high temperature. They could also compose other fatty hydrocarbons, or several other kinds of smaller molecules, as they are cooled.

Chemically bonded into a lipid molecule, the position states of these atoms fix the quantum state of the position coordinates q—a complex state specification indeed. Next, consider the momentum coordinates. p bearing, first of all, the translational and rotational states of the center of mass, then the vibrational, rotational, and bending modes of the many internal degrees of freedom, an equally complex state specification. Nonetheless, all together they proclaim "a lipid molecule in a certain position in space and time in the universe with a certain momentum." The chemically bonded position states write "lipid" in a language of atomic letters and molecular sub-unit words. The momentum states write "moving lipid" in the same language. The lipid as whole, both in structure and motion, can be regarded as matter formed by and manifesting information written in a chemical language.

Brain and Mind in Natural Computation

For human beings (as well as higher animals), the natural computation of an individual mind is located in the brain and nervous system. (Those 1 billion neurons, thousand billion synapses, and rich network of chemical circuits in which they reside are up to the task of supporting the basic operation of the mind, including the learned behaviors held in that generaously large prefrontal cortex. The neural structures of the brain encode information beyond the fundamental encoding of the dynamics of the elementary particles that compose its matter. [41, 42, 43] *

*Important brain regions implicated in its ability to encode mathematical concepts and operations include the fusiform gyrus and the intraparietal sulcus. Hippocampal frontal circuits execute

(See "A Coding Theorem," Chapter 6.) Neurons and their dynamics encode these mental states as directed by their governing Hamiltonians. Individual brains and minds form the collective mind in the interacting society of individuals and their productions: a culture or "collective consciousness."

Consider an individual mind in the social milieu of a community of minds and culture. The concept of the Turing machine, however stimulated by ideas circulating at the time of "what it means to compute, first emerged in the mind of Alan Turing. Turing's concept of a sequence of discrete machine states and a tape with cells that can be read, written to, re-written to, and moved, according to the machine state and an evolving code on the tape was enormously prescient. The motion,

$$\textit{observe-compute-move-repeat-} \cdots,$$

mimics the essential operation of the underlying paradigm of natural computation present in the qubit automaton (Chapter 3 Chapter 6 of Quantum Automata)

Principle (Turing Machine in Brain and Mind)**.** *The Turing machine running as an individual's mental structure in the mind is physically instantiated in the biology of the brain and associated neural circuits. Computations executed in brains and minds obey Landauer's principle: the information is encoded in matter—neural cells and chemical networks. Socially, individual brains extend into culture and networks of minds that make a community of mathematicians, and beyond, a wider culture. It is accurate to say a human being executing the steps of a Turing Machine in his or her mind (likely with the help of pencil and paper to off-load intermediate results) has a Turing machine running in the mind (see the fine illustration of Roger Penrose [27].)*

To which I would add, without that brain and mind, it cannot exist (except in yet other brains and minds and the extended culture they create).

A Coding Theorem

The encoding of information in computers is a familiar operation. A hardware register of some construction based on a physical effect (transistor semiconductor levels, atomic spins, etc.) holds the information. Let us say the bits in a particular hardware register are set to encode the letter "A" in some code, say an 8 bit register in ASCII code. When the bits in the register are set to 01000001, they encode the letter "A." But these bits also represent a state of nature, a set of atoms in a containing structure with its electronic states or magnetic spins in a certain state of generalized positions and momenta ($\mathsf{q}s$, and $\mathsf{p}s$). This is the "natural" state of

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the formation of associative memories and the binding of new and old information in executing mathematical thought.

the physical matter independent of our regarding them as code for a letter of the alphabet. Yet this same set of electronic levels or spin states also represents the letter state A according to the encoding design of the register. How can the same register encode two quite different states, say transistor levels or physical spins on the one hand, and the letter "A" on the other? How does nature manage this dual representation?

The answer is in the dynamics that set the register according to a Hamiltonian designed to encode the "A". The particular design Hamiltonian, for example, acts on a register in a standard state (say all bits set to "0") and dynamically advances them to the bit pattern 01000001 for "A" in the time-evolution operator. The dynamic of motion, the action of the Hamiltonian in advancing the states from the standard state to the encoded "A" *is the action of encoding*. has set up a correspondence between the physical states of the transistor or spins and the code of the letter "A". Dynamic processes of nature yoked to the design of a computer register bring forth the coincidence of a particular set of physical spins and the letter "A".

This encoding process is fundamental to nature's capacity for agents to form *representations* of other agent-states in their own physical states. I believe this dynamic process underlies all manner of "representation" in nature and is worthy of further exploration. The Hamiltonian that executes this process is the critical part of the encoding. It can only be articulated in an evolved understanding of the way quanta individuate into coherent and complex structures in condensed matter. (See Chapter 10 of AQ).

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